Math HL Name: KEY

Vectors – Chap 11 Review Period: _____

1. The vectors $\boldsymbol{u}$, $\boldsymbol{v}$ are given by $\boldsymbol{u} = 3\boldsymbol{i} + 5\boldsymbol{j} + \boldsymbol{k}$, $\boldsymbol{v} = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$.

a.) Find the unit vectors in the same directions as $\boldsymbol{u}$ and $\boldsymbol{v}$.

$$u = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix} \quad |u| = \sqrt{9+25+1} = \sqrt{35}$$

$$v = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \quad |v| = \sqrt{1+4+9} = \sqrt{14}$$

UNIT VECTORS $\begin{cases} a = \boxed{\dfrac{1}{\sqrt{35}} \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}} & \text{SAME DIRECTION AS } u \\ b = \boxed{\dfrac{1}{\sqrt{14}} \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}} & \text{SAME DIRECTION AS } v \end{cases}$

b.) Find $u \times v$

$$u \times v = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 15-(-2) \\ 1-9 \\ -6-5 \end{pmatrix} = \boxed{\begin{pmatrix} 17 \\ -8 \\ -11 \end{pmatrix}}$$

3 1

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2. Consider the points A(1, 2, 1), B(0, –1, 2), C(1, 0, 2) and D(2, –1, –6).

(a) Calculate $\overrightarrow{AB} \times \overrightarrow{BC}$.

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -3-(-2) \\ 0-(-1) \\ 2-0 \end{pmatrix} = \boxed{\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}}$$

-1 0

(b) Hence, or otherwise find the area of triangle ABC.

$$A = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}\sqrt{1+1+4} = \boxed{\frac{\sqrt{6}}{2}}$$

(c) Find the Cartesian equation of the plane P containing the points A, B and C.

NORMAL $= n = \overrightarrow{AB} \times \overrightarrow{AC}$

$n = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$

$-1x + 1y + 2z = D$

$A(1,2,1) \quad -1(1) + 1(2) + 2(1) = D$

$D = 3$

$\boxed{-1x + 1y + 2z = 3 \quad \text{OR} \quad 1x - 1y - 2z = -3}$

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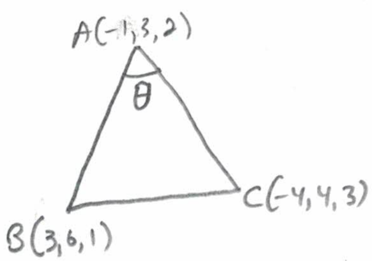
3. A triangle has its vertices at A(–1, 3, 2), B(3, 6, 1) and C(–4, 4, 3). Find $m\angle BAC$.



$$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \quad \overrightarrow{AC} = \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix}$$

$$|\overrightarrow{AB}| = \sqrt{16+9+1} = \sqrt{26}$$

$$|\overrightarrow{AC}| = \sqrt{9+1+1} = \sqrt{11}$$

$$\overrightarrow{AB} \cdot \overrightarrow{AC} = -12+3-1 = -10$$

$$\theta = \arccos\left(\frac{-10}{\sqrt{26}\sqrt{11}}\right)$$

$$\boxed{\theta = 126.3^\circ}$$

126.2501952...

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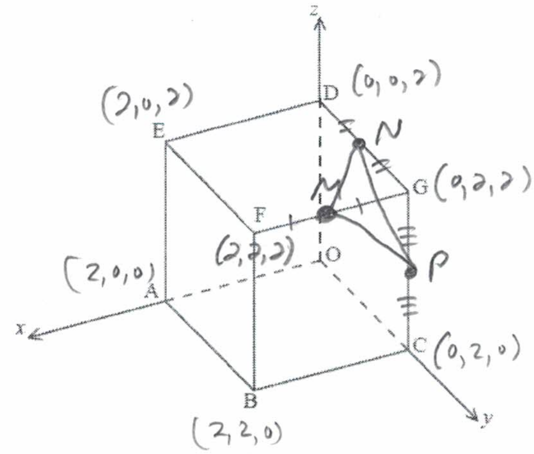
4. The diagram shows a cube OABCDEFG.

Let O be the origin, (OA) the x-axis, (OC) the y-axis and (OD) the z-axis. Let M, N and P be the midpoints of [FG], [DG] and [CG], respectively. The coordinates of F are (2, 2, 2).



(a) Find the position vectors $\overrightarrow{OM}$, $\overrightarrow{ON}$ and $\overrightarrow{OP}$ in component form.

$\overrightarrow{OM} = \boxed{\begin{pmatrix}1\\2\\2\end{pmatrix}}$ $\quad \overrightarrow{ON} = \boxed{\begin{pmatrix}0\\1\\2\end{pmatrix}}$ $\quad \overrightarrow{OP} = \boxed{\begin{pmatrix}0\\2\\1\end{pmatrix}}$

(b) Find $\overrightarrow{MP} \times \overrightarrow{MN}$.

$$\overrightarrow{MP} \times \overrightarrow{MN} = \begin{pmatrix}-1\\0\\-1\end{pmatrix} \times \begin{pmatrix}-1\\-1\\0\end{pmatrix} = \begin{pmatrix}0-1\\1-0\\1-0\end{pmatrix} = \boxed{\begin{pmatrix}-1\\1\\1\end{pmatrix}}$$

(c) **Hence,**

(i) calculate the area of the triangle MNP;

$$A = \frac{1}{2}\left|\overrightarrow{MP} \times \overrightarrow{MN}\right| = \frac{1}{2}\sqrt{1+1+1} = \boxed{\frac{\sqrt{3}}{2}}$$

(ii) show that the line (AG) is perpendicular to the plane MNP;

$$\overleftrightarrow{AG} = \begin{pmatrix}2\\0\\0\end{pmatrix} + a\begin{pmatrix}-2\\2\\2\end{pmatrix} \qquad \boxed{\begin{pmatrix}-2\\2\\2\end{pmatrix} = 2\begin{pmatrix}-1\\1\\1\end{pmatrix}}$$

$\overleftrightarrow{AG}$ PARALLEL TO THE VECTOR NORMAL TO THE PLANE MNP.

(iii) find the equation of the plane MNP.

$$-1x + 1y + 1z = D$$

$M(1,2,2)$ $\quad -1(1) + 1(2) + 1(2) = D \rightarrow D = 3$

$$\boxed{-x + y + z = 3 \quad \text{or} \quad x - y - z = -3}$$

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5. The angle between the vector $\boldsymbol{a} = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ and the vector $\boldsymbol{b} = 3\boldsymbol{i} - 2\boldsymbol{j} + m\boldsymbol{k}$ is 30°. Find the values of m.

$$a = \begin{pmatrix}1\\-2\\3\end{pmatrix} \qquad b = \begin{pmatrix}3\\-2\\m\end{pmatrix}$$

$a \cdot b = 3 + 4 + 3m = 3m + 7$

$|a| = \sqrt{1+4+9} = \sqrt{14}$

$|b| = \sqrt{9+4+m^2} = \sqrt{m^2+13}$

$$\cos 30° = \frac{3m+7}{\sqrt{14}\sqrt{m^2+13}}$$

$$\frac{\sqrt{3}}{2} \times \frac{3m+7}{\sqrt{14}\sqrt{m^2+13}}$$

$$\sqrt{42(m^2+13)} = 6m + 14$$

$$42(m^2+13) = 36m^2 + 168m + 196$$

$$\frac{6m^2 - 168m + 350 = 0}{2}$$

$$3m^2 - 84m + 175 = 0$$

$$m = \frac{84 \pm \sqrt{84^2 - 4(3)(175)}}{6}$$

$$\boxed{m = 2.27,\ 25.7}$$
