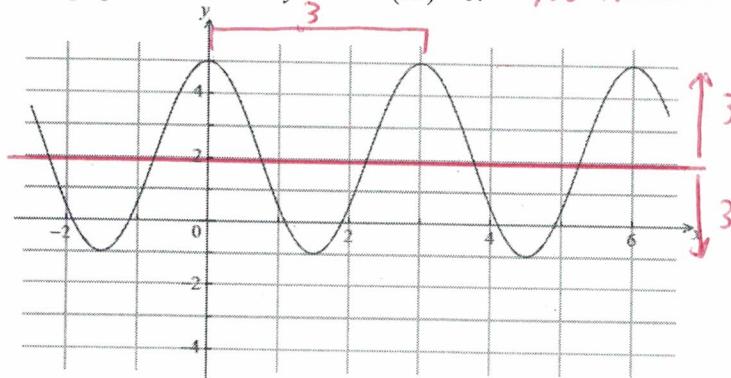


1. The graph below shows  $y = a \cos(bx) + c$ . NO HORIZONTAL SHIFT



$$a = 3$$

$$c = 2$$

$$\text{PERIOD} = \frac{2\pi}{b}$$

$$3 = \frac{2\pi}{b}$$

$$b = \frac{2\pi}{3}$$

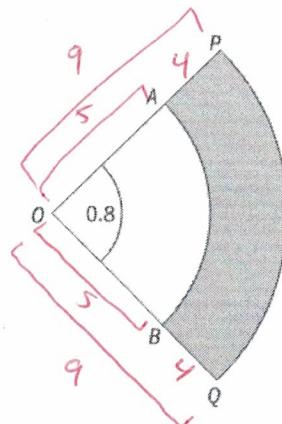
Find the values of  $a$ ,  $b$ , and  $c$ .

$$a = 3 \quad b = \frac{2\pi}{3} \quad c = 2$$

2. The diagram shows an aerial view of a swimming pool,  $ABQP$ , formed by two sectors of a circle. The angle  $\angle AOB$  is  $0.8$  radians. Find the surface area and the perimeter of the pool.

$$A = \frac{1}{2}(0.8)(9)^2 - \frac{1}{2}(0.8)(5)^2 = [22.4 \text{ m}^2]$$

$$P = 0.8(5) + 0.8(9) + 4 + 4 = [19.2 \text{ m}]$$



$$OA = 5 \text{ m}$$

$$OP = 9 \text{ m}$$

3. Find all the values of  $x$  in the interval  $[0, 2\pi]$  which satisfy the equation below.

$$3\sin x = 2\sin 2x$$

$$3\sin x = 2 \cdot 2\sin x \cos x$$

$$3\sin x - 4\sin x \cos x = 0$$

$$\sin x(3 - 4\cos x) = 0$$

$$\sin x = 0 \quad \cos x = \frac{3}{4}$$

$$\sin x = 0$$

$$x = 0^\circ, 180^\circ, 360^\circ$$

$$x = 0, \pi, 2\pi$$

$$\cos x = \frac{3}{4}$$

$$x = \cos^{-1}\left(\frac{3}{4}\right)$$

$$x = 41.4^\circ, 318.6^\circ$$

$$x = 0.723, 5.560$$

$-C + S$

DEC  
2nd

4. Given that  $\tan 2\theta = \frac{3}{4}$ , find the possible values of  $\tan \theta$ .

$$\text{Double angle identities} \quad \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\frac{2 + \tan \theta}{1 - \tan^2 \theta} = \frac{3}{4} \quad \text{CROSS MULTIPLY}$$

$$8 + 8 \tan \theta = 3 - 3 \tan^2 \theta$$

$$3 \tan^2 \theta + 8 \tan \theta - 3 = 0$$

$$(3 \tan \theta - 1)(\tan \theta + 3) = 0$$

$$\boxed{\tan \theta = \frac{1}{3}, -3}$$

5. The angle  $\theta$  satisfies the equation  $\tan \theta + \cot \theta = 3$ , where  $\theta$  is in degrees. Find all the possible values of  $\theta$  lying in the interval  $[0^\circ, 90^\circ]$ .

$$\tan \theta + \cot \theta = 3$$

$$\left( \tan \theta + \frac{1}{\tan \theta} = 3 \right) \cdot \tan \theta$$

$$\tan^2 \theta + 1 = 3 \tan \theta$$

$$\tan^2 \theta - 3 \tan \theta + 1 = 0$$

$$\tan \theta = \frac{3 \pm \sqrt{9 - 4(1)(1)}}{2}$$

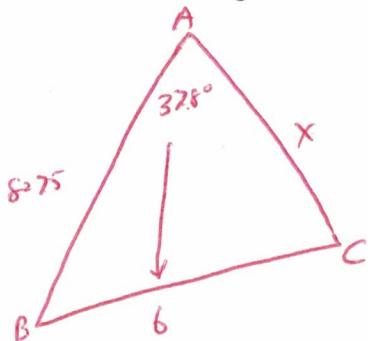
$$\tan \theta = \frac{3 \pm \sqrt{5}}{2}$$

$$\tan \theta = 2.618 \text{ or } 0.381966$$

$$\boxed{\theta = 69.1^\circ, 20.9^\circ}$$

Quadrant I only!

6. Consider triangle ABC with  $\angle BAC = 37.8^\circ$ ,  $AB = 8.75$ , and  $BC = 6$ . Find AC.



$$\frac{\sin 37.8^\circ}{6} = \frac{\sin \angle C}{8.75}$$

$$\sin \angle C = \frac{8.75 \sin 37.8^\circ}{6}$$

$$\sin \angle C = 0.893822786577$$

$$\angle C = 63.3576185324$$

$$\angle C = 116.642381468$$

$$\frac{\sin 32.8}{6} = \frac{\sin 78.8}{x}$$

$$\boxed{x = 9.6}$$

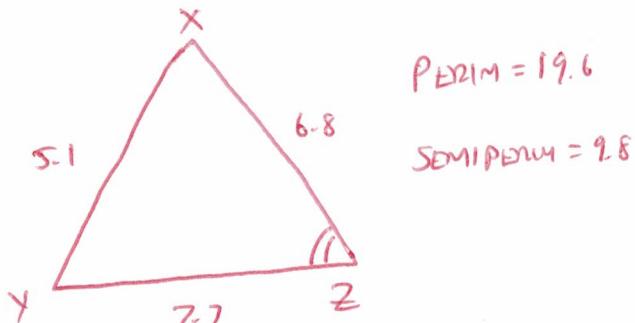
$$\frac{\sin 32.8}{6} = \frac{\sin 25.6}{x}$$

$$\boxed{x = 4.2}$$

$$\angle B = 78.8423814676$$

$$\angle B = 25.5576185324$$

7. Find the area of triangle XYZ to the nearest tenth of a unit if XY = 5.1, YZ = 7.7, and XZ = 6.8.



METHOD 1:  $A = \sqrt{9.8(9.8 - 5.1)(9.8 - 7.7)(9.8 - 6.8)}$

$$A = \sqrt{9.8(4.7)(2.1)(3)}$$

$\boxed{A = 17.0}$  17.0346118242

METHOD 2

$$5.1^2 = 7.7^2 + 6.8^2 - 2(7.7)(6.8) \cos \angle Z$$

$$\cos \angle Z = \frac{7.7^2 + 6.8^2 - 5.1^2}{2(7.7)(6.8)}$$

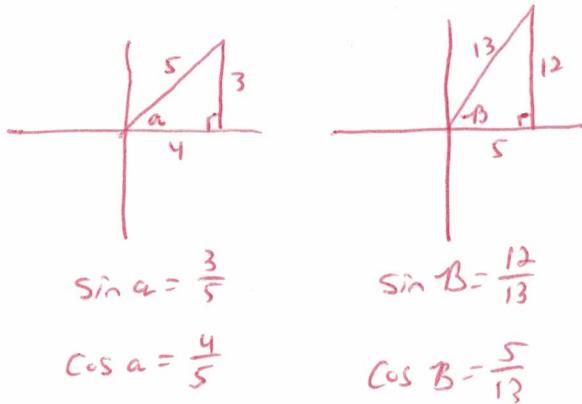
$$\cos \angle Z = 0.75935828877$$

$$\angle Z = 0.708470007495 \text{ RADIANS}$$

$$A = \frac{1}{2}(7.7)(6.8) \sin(0.708)$$

$\boxed{A = 17.0}$

8. Given that  $\sin \alpha = \frac{3}{5}$  and  $\sin \beta = \frac{12}{13}$ , where both  $\alpha$  and  $\beta$  are acute angles, find the exact value of  $\sin(\alpha + \beta)$ .



Quadrant I

$$\begin{aligned} \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \sin \beta \cos \alpha \\ &= \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{12}{13}\right)\left(\frac{4}{5}\right) \\ &= \frac{15}{65} + \frac{48}{65} \\ &= \boxed{\frac{63}{65}} \end{aligned}$$

9. Verify the following identities:  $\frac{1}{1+\cos \theta} + \frac{1}{1-\cos \theta} = \frac{2}{\sin^2 \theta}$ .

$$\frac{1-\cos \theta}{(1+\cos \theta)(1-\cos \theta)} + \frac{1+\cos \theta}{(1-\cos \theta)(1+\cos \theta)} = \frac{2}{\sin^2 \theta}$$

$$\frac{2}{1-\cos^2 \theta} = \frac{2}{\sin^2 \theta}$$

$\boxed{\frac{2}{\sin^2 \theta} = \frac{2}{\sin^2 \theta}}$

10. Express  $\frac{1+\tan^2 x}{\cos^2 x}$  in terms of  $\sin x$ , simplifying your answer.

PYTH. IDENTITY

$$\frac{\sec^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \cdot \frac{1}{\cos^2 x} = \frac{1}{1-\sin^2 x} \cdot \frac{1}{1-\sin^2 x} = \boxed{\frac{1}{1-2\sin^2 x + \sin^4 x}}$$

11. Solve  $2(\ln x)^2 - 3(\ln x) + 1 = 0$  for  $x$ . Give your answers in exact form (**no decimal approximations**).

$$\begin{aligned} 2(\ln x)^2 - 3(\ln x) + 1 &= 0 \\ 2y^2 - 3y + 1 &= 0 \\ (2y-1)(y-1) &= 0 \\ (2\ln x - 1)(\ln x - 1) &= 0 \end{aligned}$$

$2\ln x = 1 \quad \ln x = 1$   
 $\ln x = \frac{1}{2} \quad \ln x = 1$   
 $x = e^{\frac{1}{2}} \quad x = e^1$   
 $x = e^{\frac{1}{2}}, e$

12. Solve  $\log(x^2 + 1) = 1 + 2 \log x$ .

$$\begin{aligned} \log_{10}(x^2 + 1) &= \log_{10} 10 + 2 \log_{10} x \\ \log(x^2 + 1) &= \log(10x^2) \\ x^2 + 1 &= 10x^2 \\ 9x^2 &= 1 \\ x^2 &= \frac{1}{9} \\ x &= \pm \frac{1}{3} \end{aligned}$$

$\boxed{x = \frac{1}{3}}$

**ALT. METHOD**  
 $\log(x^2 + 1) - \log x^2 = 1$   
 $\log\left(\frac{x^2 + 1}{x^2}\right) = 1$   
 $10^1 = \frac{x^2 + 1}{x^2}$  BASE TO ANSWER POWER  
 $= \text{NUMBER OF LOG OF}$   
 $10x^2 = x^2 + 1$   
 $9x^2 = 1$   
 $\boxed{x = \frac{1}{3}}$

13. Solve the following equations for  $x \in \mathbb{R}$ .

$$\begin{aligned} \text{a.) } 9^x &= 12(3^x) - 27 \\ (3^2)^x &= 12(3^x) - 27 \\ (3^x)^2 - 12(3^x) + 27 &= 0 \\ (3^x - 9)(3^x - 3) &= 0 \\ 3^x - 9 &= 0 \quad 3^x - 3 = 0 \\ 3^x &= 9 \quad 3^x = 3 \\ \boxed{x = 2, x = 1} \end{aligned}$$

$$\begin{aligned} \text{b.) } 3^x - \frac{9}{3^x} &= 8 \\ (3^x)^2 - 9 &= 8(3^x) \\ (3^x)^2 - 8(3^x) - 9 &= 0 \\ (3^x - 9)(3^x + 1) &= 0 \\ 3^x - 9 &= 0 \quad 3^x + 1 = 0 \\ 3^x &= 9 \quad 3^x = -1 \\ \boxed{x = 2} \end{aligned}$$

No Sol.