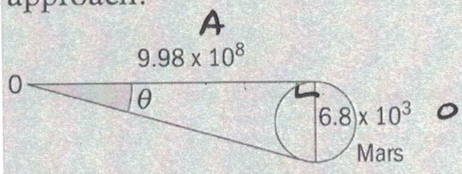
Math HL – Year 1
Trigonometry – Introduction

Name: ______________________

Period: _____

A planet is in opposition when it is directly opposite the Sun from our viewpoint on Earth. During opposition in March 2012 Mars is closer to the Earth than at any other time at a distance of approximately 9.98×10^8 km. Mars has an approximate diameter of 6.8×10^3 km. What is the angle subtended by Mars when viewed during closest approach?



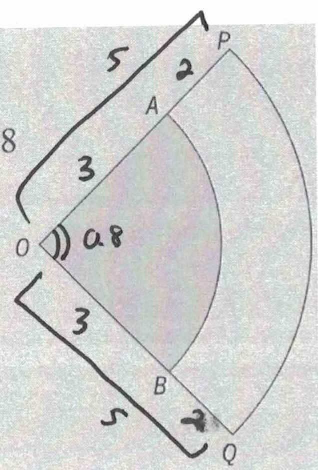
$$\tan\theta = \frac{OPP}{ADJ}$$

$$\tan\theta = \frac{6.8 \times 10^3}{9.98 \times 10^8}$$

$$\theta = \tan^{-1}\left(\frac{6.8 \times 10^3}{9.98 \times 10^8}\right)$$

$\theta = 6.81 \times 10^{-6}$ RADIANS $\quad$ (3.90×10^{-4} DEGREES)

The diagram shows two arcs which subtend the same angle.
Given that $OA = 3$ cm, $OP = 5$ cm and $\angle AOB = 0.8$ radians, find the area which is **not** shaded and its perimeter.



OUTER

$A = \frac{1}{2}(0.8)(5)^2$

$A = 10$

INNER

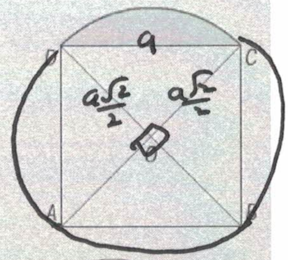
$A = \frac{1}{2}(0.8)(3)^2$

$A = 3.6$

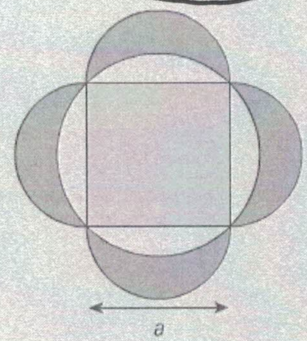
$A_{\text{NOT SHADED}} = 10 - 3.6 = \boxed{6.4\text{ cm}^2}$

$P = 2 + 2 + 0.8(3) + 0.8(5) = \boxed{10.4\text{ cm}}$

a The diagram shows a square *ABCD* with sides *a* cm long which just fits inside a circle. Find the area of the shaded segment.



b On each of the sides of the square, semicircles are drawn to form four crescents. Show that the area shaded in dark grey is equal to the area of the square .



a.) $A = \text{SECTOR AREA} - \text{ISOS. } \triangle$

$$= \frac{1}{2}\cdot\frac{\pi}{2}\cdot\left(\frac{a\sqrt{2}}{2}\right)^2 - \frac{1}{2}\left(\frac{a\sqrt{2}}{2}\right)\left(\frac{a\sqrt{2}}{2}\right)$$

$$= \left(\frac{\pi}{4}\cdot\frac{2a^2}{4}\right) - \left(\frac{1}{2}\cdot\frac{2a^2}{4}\right)$$

$$= \frac{\pi a^2}{8} - \frac{2a^2}{8}$$

$$\boxed{A = \frac{\pi a^2 - 2a^2}{8} \text{ or } \frac{a^2(\pi-2)}{8}}$$

b.) $A_{SQUARE} = \boxed{a^2}$

$$A_{OUTER} = \frac{1}{2}\cdot\pi\left(\frac{a}{2}\right)^2 - (\text{ANSWER FROM (a)})$$

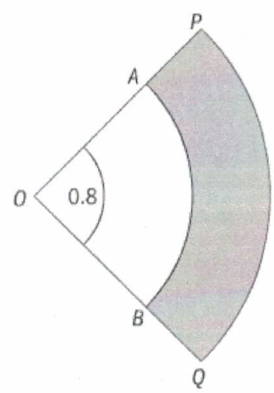
$$= \frac{\pi}{2}\cdot\frac{a^2}{4} - \left(\frac{\pi a^2 - 2a^2}{8}\right)$$

$$= \frac{\pi a^2}{8} - \frac{\pi a^2}{8} + \frac{2a^2}{8}$$

$$= \frac{a^2}{4}$$

$$4 \text{ PARTS} = 4\cdot\frac{a^2}{4} = \boxed{a^2} \rightarrow \boxed{a^2 = a^2}$$

2 The diagram shows an aerial view of a swimming pool, *ABQP*, formed by two sectors of a circle. The angle *AOB* is 0.8 radians. Find the surface area and the perimeter of the pool.

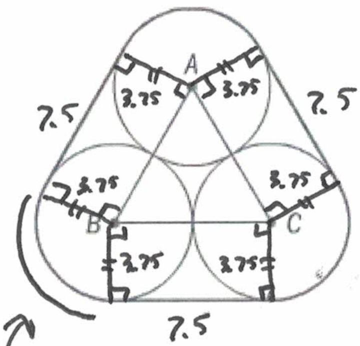


$OA = 5\text{ m}$
$OP = 9\text{ m}$

$$A = \frac{1}{2}(0.8)(9)^2 - \frac{1}{2}(0.8)(5)^2 = \boxed{22.4\text{ m}^2}$$

$$P = 0.8(5) + 0.8(9) + 4 + 4 = \boxed{19.2\text{ m}}$$

In a special offer, three cans of cat food are sold for the price of two. The cans are wrapped by a plastic foil as shown in the diagram. Each can has a diameter of 7.5 cm.
Find the length of plastic foil required to hold the cans together.
(Assume that no overlapping is required.)



$d = 7.5$

$r = 3.75$

$L = \theta r$

$L = \left(\frac{2\pi}{3}\right)(3.75)$

$L = \frac{2\pi}{3} \cdot \frac{15}{4}$

$L = \frac{5\pi}{2}$

TOTAL LENGTH $= 7.5 + 7.5 + 7.5 + \frac{5\pi}{2} + \frac{5\pi}{2} + \frac{5\pi}{2}$

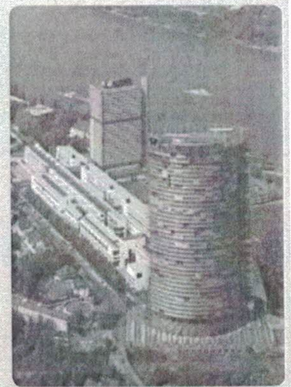
$= 22.5 + \frac{15\pi}{2}$

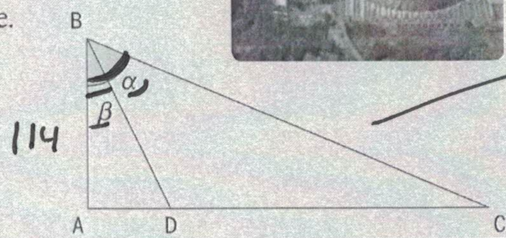
$= 22.5 + 7.5\pi$

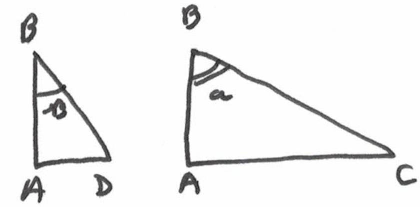
$\approx \boxed{46.062 \text{ cm}}$

The photo shows the river Rhine and the building known as Langer Eugen in Bonn which houses the United Nations. The building is 114 m high.

The angles α and β from the top of the tower to the edges of the river are measured. Given that $\alpha = 75°$ and $\beta = 19°$, calculate the width of the river, DC, giving your answer to the nearest metre.







NO NEED TO CHANGE TO RADIANS ON THIS PROBLEM!!

$\tan B = \frac{AD}{114}$

$AD = 114 \cdot \tan 19°$

$\tan a = \frac{AC}{114}$

$AC = 114 \cdot \tan 75°$

$BC = AC - AD = 114 \cdot \tan 75° - 114 \tan 19° = \boxed{386.2\text{ m}}$

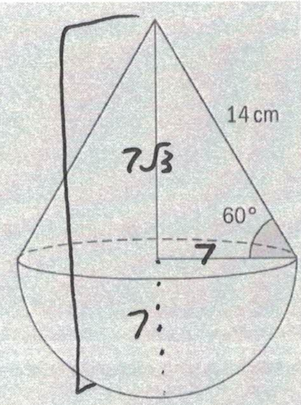
$= \boxed{386\text{ m}}$

NEAREST METER

The solid below is made up of a cone and a hemisphere.

a Find the height of the solid.

b Show that the surface area of the hemispherical base is equal to the curved surface area of the cone.



a.) 30-60-90

$x \quad x\sqrt{3} \quad 2x$

$7 \quad 7\sqrt{3} \quad 14$

$\boxed{h = 7\sqrt{3} + 7}$

b.) $SA_{SPHERE} = \frac{1}{2} \cdot 4\pi r^2 = \frac{1}{2} \cdot 4\pi (7)^2 = \boxed{98\pi}$

$SA_{CONE} = \pi r l = \pi (7)(14) = \boxed{98\pi}$

$= \checkmark$