Trig Graphs - Applications (1 per sheet)

1. The temperature in an office is controlled by an electronic thermostat. The temperatures vary according to the sinusoidal function:

$$y = 19 + 6\sin\left(\frac{\pi}{12}(x-11)\right)$$

where y is the temperature (°C) and x is the time in hours past midnight.

a.) What is the temperature in the office at 9 A.M. when employees come to work?

b.) What are the maximum and minimum temperatures in the office?

$$y = 6\sin\left(\frac{\pi}{12}(x-11)\right) + 19$$

$a = 6$

$b = \frac{\pi}{12}$ PERIOD $= \frac{2\pi}{\frac{\pi}{12}} = 24$

$c = 11$

$d = 19$

x	y
11	19
17	25
23	19
29	13
35	19

$\xrightarrow{-24 \text{ hrs}}$

x	y	
-13	19	
-7	25	MAX
-1	19	
5	13	MIN
11	19	

a.) $f(9) = 6\sin\left(\frac{\pi}{12}(9-11)\right) + 19$

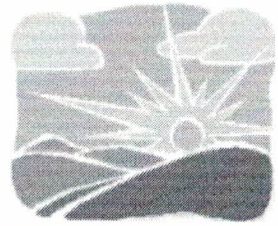
$= 6\sin\left(-\frac{\pi}{6}\right) + 19$

$= 6\left(-\frac{1}{2}\right) + 19$

$= \boxed{16°C}$

b.) $\boxed{\begin{array}{l} MAX = 25°C \\ MIN = 13°C \end{array}}$

2. The number of hours of daylight measured in one year in Ellenville can be modeled by a sinusoidal function. During 2006, (not a leap year), the longest day occurred on June 21 with 15.7 hours of daylight. The shortest day of the year occurred on December 21 with 8.3 hours of daylight. Write a sinusoidal equation to model the hours of daylight in Ellenville.



DAYS IN YEAR = 365 PERIOD = $365 = \frac{2\pi}{b}$ $\quad \underline{b = \frac{2\pi}{365}}$

DAYS IN EACH MONTH (NON-LEAP YEAR)

J	F	M	A	M	J	J	A	S	O	N	D
31	28	31	30	31	30	31	31	30	31	30	31

151

PEAK = 15.7

VALLEY = 8.3

$\frac{15.7+8.3}{2} = \frac{24}{2} = 12 \quad \underline{d = 12}$

$15.7 - 12 = 3.7 \quad \underline{a = 3.7}$

$\frac{365}{4} = 91.25$ (WHERE FIRST PEAK SHOULD OCCUR)

JUNE 21 = DAY 172

DEC 21 = DAY 355

SHOULD BE ½ PERIOD ($365/2 = 182.5$)

BUT IS SLIGHTLY OFF ($355 - 172 = \underline{183}$)

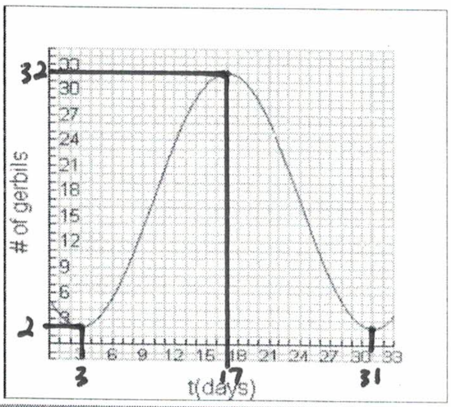
$172 - 91.25 = 80.75 \quad \underline{C = 80.75}\ (\longrightarrow)$

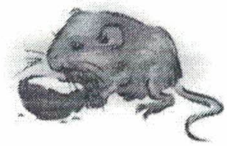
$$\boxed{Y = 3.7 \sin \frac{2\pi}{365}\left(x - \underline{80.75}\right) + 12}$$

OTHER VALUES ARE ACCEPTABLE (SUCH AS 81)

Trig Graphs - Applications (1 per sheet)

3. A pet store clerk noticed that the population in the gerbil habitat varied sinusoidally with respect to time, in days. He carefully collected data and graphed his resulting equation. From the graph, determine the amplitude, period, horizontal shift and vertical shift. Write the equation of the graph.





PEAK @ (17, 32)

VALLEYS @ (3, 2) AND (31, 2)

$\frac{32+2}{2} = \frac{34}{2} = 17$ $\quad \underline{d = 17}$

$32 - 17 = 15$ $\quad \underline{a = 15}$

$31 - 3 = 28$ $\quad$ PERIOD $= 28 = \frac{2\pi}{b}$ $\quad \underline{b = \frac{\pi}{14}}$

$28/4 = 7$ (WHERE FIRST PEAK SHOULD OCCUR)

$17 - 7 = 10$ $\quad \underline{c = 10}$ $(\rightarrow)$

$a = 15$

PERIOD $= 28$

$c = 10 \, (\rightarrow)$

$d = 17$

$\boxed{y = 15 \sin \frac{\pi}{14}(x - 10) + 17}$

or

$y = 15 \cos \frac{\pi}{14}(x - 17) + 17$

4. Given the following equations, determine the amplitude, period, horizontal shift, and vertical shift of each equation.

$$y = 2\sin\left(\frac{\pi}{3}(x-2)\right) - 4$$

$$y = -4 + 2\sin\left(\frac{\pi}{3}(x-3.5)\right)$$

Are these two equations equivalent?
Support your answer graphically and algebraically.

(A) $y = 2\sin\frac{\pi}{3}(x-2) - 4$

$a = 2$

$b = \frac{\pi}{3}$ PERIOD $= \frac{2\pi}{\frac{\pi}{3}} = 6$

$c = 2$ $(\longrightarrow)$

$d = -4$

(B) $y = 2\sin\frac{\pi}{3}(x-3.5) - 4$

$a = 2$

$b = \frac{\pi}{3}$ PERIOD $= 6$

$c = 3.5$ $(\longrightarrow)$

$d = -4$

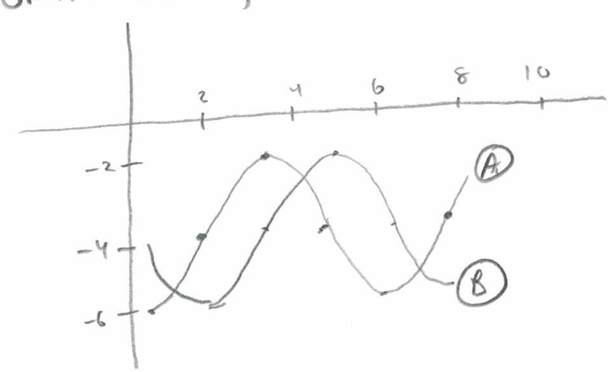
EQUATIONS ARE <u>NOT</u> EQUIVALENT.

(A) $f(2) = 2\sin\frac{\pi}{3}\underbrace{(2-2)}_{0} - 4 = -4$

(B) $f(2) = 2\sin\frac{\pi}{3}(-1.5) - 4 \neq -4$

NOT =

GRAPHICALLY, IDENTICAL a, b, d, BUT w/ DIFFERENT C.



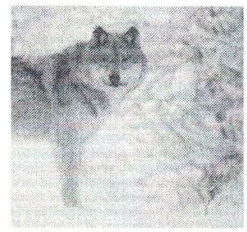
IF (B) WAS COS,
THE EQUATIONS WOULD
BE EQUIVALENT.

Trig Graphs - Applications (1 per sheet)

5. Environmentalists use sinusoidal functions to model populations of predators and prey in the environment. In a particular study, the population of rabbits was modeled by the function

$$R(x) = 15{,}000\sin\left(\frac{\pi}{2}\left(x+\frac{\pi}{2}\right)\right)+25{,}000$$

The population of wolves in the same environmental area was modeled by the function



$$W(x) = 2{,}000\sin\left(\frac{\pi}{2}x\right)+5{,}000$$

In each formula, x represents time in months.

Using the graphs of these two equations, make a statement regarding the relationship between the number of rabbits and the number of wolves in this environmental area.

$R(x)$

$a = 15000$

$b = \frac{\pi}{2}$ PERIOD $= \frac{2\pi}{\frac{\pi}{2}} = 4$

$c = \frac{\pi}{2}$ $(\leftarrow)$

$d = 25000$

MAX $= 40{,}000$ @ $x = 1 - \frac{\pi}{2}$ $(-0.571, 3.429)$

MIN $= 10{,}000$ @ $x = 3 - \frac{\pi}{2}$ $(1.429, 5.429)$

$W(x)$

$a = 2000$

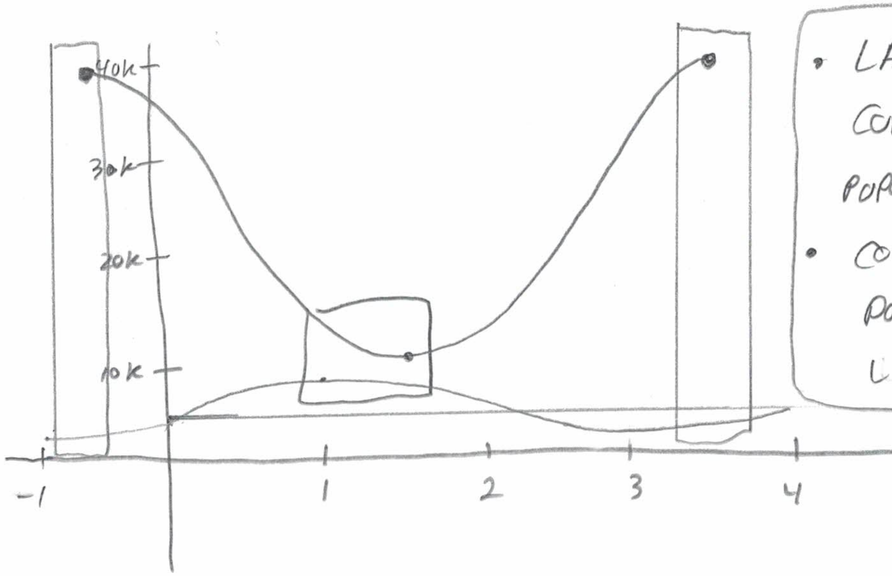
$b = \frac{\pi}{2}$ PERIOD $= \frac{2\pi}{\frac{\pi}{2}} = 4$

$c = 0$

$d = 5000$

MAX $= 7{,}000$ @ $x = 1$

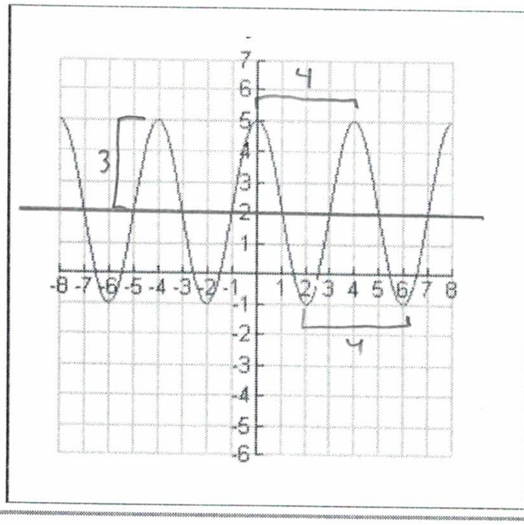
MIN $= 3{,}000$ @ $x = 3$

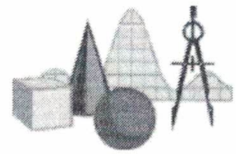


- LARGE RABBIT POPULATIONS CORRESPOND TO SMALL WOLF POPULATIONS.
- CONVERSELY, SMALLER RABBIT POPULATIONS CORRESPOND TO LARGE WOLF POPULATIONS.

Trig Graphs - Applications (1 per sheet)

6. Write both a sine and a cosine equation for the following graph.





MAX = 5 $\quad \frac{5+(-1)}{2} = \frac{4}{2} = 2 \quad \underline{d=2}$

MIN = −1 $\quad 5-2=3 \quad \underline{a=3}$

PERIOD = 4 $\quad \frac{2\pi}{b} = 4 \quad \underline{b = \frac{\pi}{2}}$

sin → FIRST PEAK SHOULD OCCUR AT 1,

SO C = −1 OR C = 3

COS → FIRST PEAK SHOULD OCCUR AT 0,

SO C = 0 OR C = 4

$y = 3\sin\frac{\pi}{2}(x+1)+2$

OR

$y = 3\sin\frac{\pi}{2}(x-3)+2$

$y = 3\cos\frac{\pi}{2}x + 2$

OR

$y = 3\cos\underline{\frac{\pi}{2}(x-4)+2}$

NOT NECESSARY

Trig Graphs - Applications (1 per sheet)

7. A team of biologists have discovered a new creature in the rain forest. They note the temperature of the animal appears to vary sinusoidally over time. A maximum temperature of 125 ° occurs 15 minutes after they start their examination. A minimum temperature of 99 ° occurs 28 minutes later. The team would like to find a way to predict the animal's temperature over time in minutes. Your task is to help them by creating a graph of one full period and an equation of temperature as a function over time in minutes

MAX = 125 $\quad \frac{125+99}{2} = \frac{224}{2} = 112 \quad \underline{d = 112}$

MIN = 99

71

$125 - 112 = 13 \quad \underline{a = 13}$

MAX → MIN = 1/2 PERIOD

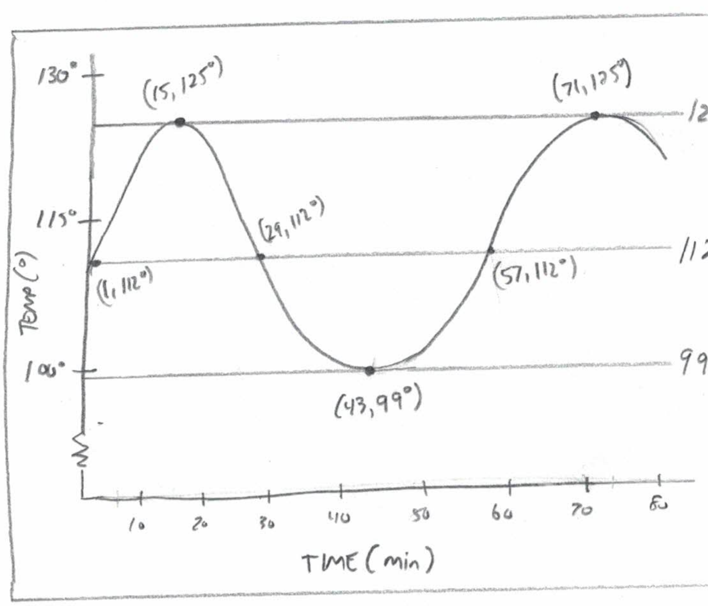
PERIOD = $28 \cdot 2 = \underline{56} \quad \frac{2\pi}{b} = 56 \quad \underline{b = \frac{\pi}{28}}$

FIRST PEAK SHOULD OCCUR AT 56/4, OR 14.

IT OCCURS AT 15.

$15 - 14 = 1 \quad c = 1$

$$\boxed{y = 13 \sin \frac{\pi}{28}(x-1) + 112}$$



Trig Graphs - Applications (1 per sheet)

8. The angle of inclination of the sun changes throughout the year. This changing angle affects the heating and cooling of buildings. The overhang of the roof of a house is designed to shade the windows for cooling in the summer and allow the sun's rays to enter the house for heating in the winter.

The sun's angle of inclination at noon in central New York state can be modeled by the formula:

$$\textbf{Angle of Inclination (in degrees)} = -23.5\cos\left(\frac{360}{365}(x+10)\right)+47$$

where x is the number of days elapsed in the day of the year, with January first represented by $x = 1$, January second represented by $x = 2$, and so on.

Find the sun's angle of inclination at noon on Valentine's Day.
Sketch a graph illustrating the changes in the sun's angle of inclination throughout the year. On what date of the year is the angle of inclination at noon the greatest in central New York state?

☆ ASSUME NON-LEAP YEAR

$$y = -23.5\cos\left(\frac{360}{365}(x+10)\right)+47$$

a.) FEB 14 = $\underset{\text{JAN}}{31} + 14$ = DAY 45

$$f(45) = -23.5\cos\left(\frac{360}{365}(45+10)\right)+47$$

$$= \boxed{33.5^\circ}$$

b.) $a = -23.5$ MAX $= 23.5 + 47 = \boxed{70.5}$ MAX... WHEN?

$d = 47$ MIN $= -23.5 + 47 = 23.5$

$b = \frac{360}{365}$ PERIOD $= \dfrac{360}{\frac{360}{365}} = 365$ ← 360° INSTEAD OF 2π

WITH A COS GRAPH, THE PEAKS SHOULD OCCUR AT 0 AND 365.
SINCE YOUR AMPLITUDE IS NEGATIVE, THE GRAPH IS REFLECTED
OVER THE X-AXIS WITH THE FIRST PEAK OCCURRING AT $\frac{365}{2} = 182.5$.

SINCE $C = -10$, $(x+10)$, WE WANT $X = 172.5$.

J	F	M	A	M	J
31	28	31	30	31	30

$\underbrace{31 + 28 + 31 + 30 + 31}_{151 \text{ DAYS}}$

$172.5 - 151 = 21.5$

$\boxed{\text{MAX ANGLE OF } 70.5^\circ \text{ OCCURS ON JUNE } 21^* \text{ (NON-LEAP YEAR)}}$

*SUMMER SOLSTICE OCCURS ON JUNE 20-22.