

Review Topic List

- Cumulative frequency diagrams (CFDs) and IQR (6C) **6C #5**
- Mean, variance (Mean of the squares minus the square of the mean), and standard deviation of a set of numbers (6D) **6D #4**
- Union and intersection ($P(A \cup B) = P(A) + P(B) - P(A \cap B)$), Venn diagrams, A vs A' (6E through rest of chapter) **6F #2, 6J #2**
- Probabilities and combinations (6G) **6G #1,4**
- Mutually exclusive ($P(A \cap B) = \emptyset$) and independent events ($P(A \cap B) = P(A) \cdot P(B)$) (6K) **6K #4,5**
- Conditional probability and tree diagrams (6J-6M) **6J #2, 6M #7,12**

1. The heights (rounded to nearest cm) of 6 randomly selected students were as follows;

176, 192, 181, 178, 188, 174

Find the mean, variance, and standard deviation of the possible outcomes.

$$\mu = \frac{1089}{6}$$

$$\boxed{\mu = 181.5}$$

$$\text{AVG SQUARES} = \frac{176^2 + 192^2 + 181^2 + 178^2 + 188^2 + 174^2}{6}$$

$$= \frac{197,905}{6}$$

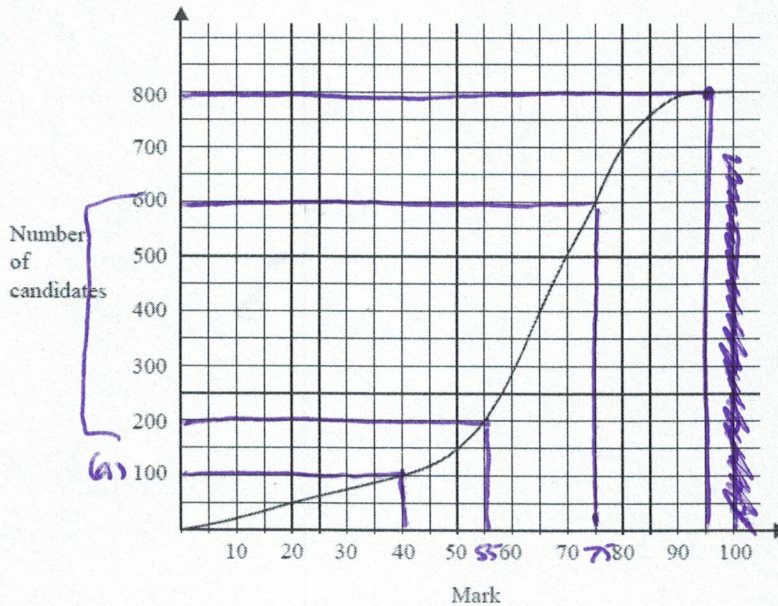
$$= 32,984.\overline{16}$$

$$\sigma^2 = 32,984.\overline{16} - 181.5^2$$

$$\boxed{\sigma^2 = 41.91\overline{6}}$$

$$\boxed{\sigma = 6.474}$$

2. A test marked out of 100 is written by 800 students. The cumulative frequency graph for the marks is given below.



(a) Write down the number of students who scored 40 marks or less on the test.

100

(b) The middle 50% of test results lie between marks a and b , where $a < b$. Find a and b .

a = 55 b = 75

(Total 6 marks)

3. (a) The events A and B are such that $P(A \cup B) = \frac{3}{5}$, $P(B) = \frac{2}{5}$, and $P(B|A) = \frac{3}{7}$
- (i) Are A and B independent? State your reason.

No. $P(B) \neq P(B|A)$ ($\frac{2}{5} \neq \frac{3}{7}$)

- (ii) Find $P(A)$.

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \quad P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$\frac{3}{7} \cdot P(A) = P(A \cap B) \rightarrow \frac{3}{7} \cdot P(A) = P(A) + \frac{2}{5} - \frac{3}{5}$$

SUBSTITUTION

$$\frac{1}{5} = \frac{4}{7} \cdot P(A)$$

$$P(A) = \frac{7}{20}$$

4. There are 20 students in a class, of which 13 are girls and 7 are boys. Five students are selected at random to form a committee. Calculate the probability that the committee contains

(a) no boys; $\frac{\binom{13}{5}}{\binom{20}{5}} = \frac{1287}{15,504} = \frac{429}{5,168}$ or $\boxed{0.0830}$

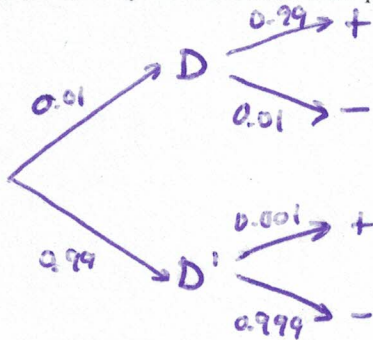
- (b) more boys than girls.

5B 06
4B 16
3B 26

$$\frac{\binom{7}{5} + \binom{7}{4}\binom{13}{1} + \binom{7}{3}\binom{13}{2}}{\binom{20}{5}} = \frac{21 + 455 + 2,730}{15,504} = \frac{3206}{15,504}$$
 or $\boxed{0.207}$

5. In a population of rabbits, 1% are known to have a particular disease. A test is developed for the disease that gives a positive result for a rabbit that **does** have the disease in 99% of cases. It is also known that the test gives a positive result for a rabbit that **does not** have the disease in 0.1% of cases. A rabbit is chosen at random from the population.

- (a) Find the probability that the rabbit tests positive for the disease.



$$P(+)= 0.01 \cdot 0.99 + 0.99 \cdot 0.001$$

$$P(+)= 0.01089$$
 or $\boxed{0.0109}$

- (b) Given that the rabbit tests positive for the disease, show that the probability that the rabbit does not have the disease is less than 10%.

$$P(D'|+) = \frac{P(D' \cap +)}{P(+)} = \frac{0.99 \cdot 0.001}{0.01089} = 0.0909$$
 or $\boxed{9.09\%}$