IB Math HL1
Unit 01 – Practice (Sequences, Series, and Factorials)

Name: KEY

Period: ____

1. An arithmetic sequence has 5 and 13 as its first two terms respectively.

(a) Write down, in terms of n, an expression for the nth term, a_n.

$a_1 = 5$ $\quad a_n = 5 + 8(n-1)$

$d = 8$ $\quad a_n = 5 + 8n - 8$

$\boxed{a_n = 8n - 3}$

(b) Find the sum of all the terms of the sequence which are less than 400.

$8n - 3 < 400$ $\quad a_1 = 5$

$8n < 403$ $\quad a_{50} = 8(50) - 3 = 397$

$n < 50.375$ $\quad S_{50} = \frac{50}{2}(5 + 397)$

50 TERMS $\quad \boxed{S_{50} = 10{,}050}$

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2. The common ratio of the terms in a geometric series is 2^x.

(a) If the first term of the series is 35, find the value of x for which the sum to infinity is 40.

$a_1 = 35$

$S = \frac{a_1}{1-r}$

$40 = \frac{35}{1-r}$

$1 - r = \frac{35}{40}\left(\frac{7}{8}\right)$

$r = \frac{1}{8}$

$2^x = \frac{1}{8}$

$\boxed{x = -3}$

(b) Find the 4th term in the series.

$a_n = 35 \cdot \left(\frac{1}{8}\right)^{n-1}$

$a_4 = 35 \cdot \left(\frac{1}{8}\right)^3$

$\boxed{a_4 = \frac{35}{512}}$ or 0.068359

(c) Find the sum of the first 5 terms.

$S_5 = \frac{35\left(1-\left(\frac{1}{8}\right)^5\right)}{1-\frac{1}{8}}$

$S_5 = \frac{35\left(1-\frac{1}{32768}\right)}{\frac{7}{8}}$

$= 40\left(\frac{32767}{32768}\right)$

$S_5 = \boxed{\frac{163835}{4096} \text{ or } 39.9988}$

3. (a) The sum of the first six terms of an arithmetic series is 81. The sum of its first eleven terms is 231. Find the first term and the common difference.

$S_n = \frac{n}{2}(a_1 + a_n)$

① $S_6 = \frac{6}{2}(a_1 + a_6) = 81$

② $S_{11} = \frac{11}{2}(a_1 + a_{11}) = 231$

$a_6 = a_1 + 5d$

$a_{11} = a_1 + 10d$

① $3(a_1 + a_1 + 5d) = 81$ ② $\frac{11}{2}(a_1 + a_1 + 10d) = 231$

$2a_1 + 5d = 27$ $2a_1 + 10d = 42$

SYSTEM OF EQUATIONS

$2a_1 + 10d = 42$

$-2a_1 - 5d = -27$

$5d = 15$

$\boxed{d = 3}$

$2a_1 + 15 = 27$

$2a_1 = 12$

$\boxed{a_1 = 6}$

CHECK ✓

a_1	a_2	a_3	a_4	a_5	a_6
6	9	12	15	18	21

(b) The sum of the first two terms of a geometric series is 1 and the sum of its first four terms is 5. If all of its terms are positive, find the first term and the common ratio.

$S_n = \frac{a_1(1-r^n)}{1-r}$

$S_2 = \frac{a_1(1-r^2)}{1-r}$ $S_4 = \frac{a_1(1-r^4)}{1-r}$

$1 = \frac{a_1(1-r^2)}{1-r}$ $5 = \frac{a_1(1-r^4)}{1-r}$

$1 = \frac{a_1(1-\cancel{r})(1+r)}{1-\cancel{r}}$ $5 = a_1\frac{(1+r^2)(1+r)\cancel{(1-r)}}{\cancel{1-r}}$

$a_1 = \frac{1}{1+r}$ $5 = \frac{1}{\cancel{1+r}} \cdot (1+r^2)\cancel{(1+r)}$

$5 = 1 + r^2$

$r^2 = 4$

$r = \pm 2$

$\boxed{r = 2}$ (ALL POSITIVE TERMS)

$1 = \frac{a_1(1-4)}{1-2}$

$1 = \frac{a_1(-3)}{-1}$

$\boxed{a_1 = \frac{1}{3}}$

4. Solve the following equations.

a.) $\frac{n!}{(n-2)!} = 20$

$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 20$

$n^2 - n - 20 = 0$

$(n-5)(n+4) = 0$

$n = 5, \cancel{-4}$

$\boxed{n = 5}$

b.) $\binom{n}{2} = 351$

$\frac{n!}{2!(n-2)!} = 351$

$\frac{n(n-1)\cancel{(n-2)!}}{2!\cancel{(n-2)!}} = 351$

$n^2 - n = 702$

$n^2 - n - 702 = 0$

$(n-27)(n+26) = 0$

$n = 27, -26$

OR

$n = \frac{1 \pm \sqrt{1^2 + 4(1)(702)}}{2}$

$n = \frac{1 \pm 53}{2}$

$n = 27, \cancel{-26}$

$\boxed{n = 27}$

5. Given the expression $\left(\frac{1}{x} - 2x^3\right)^8$;

(a) Find the coefficient of the term containing x^{12}.

$$\binom{8}{3}\left(\frac{1}{x}\right)^3\left(-2x^3\right)^5 = \frac{8\cdot7\cdot6}{3\cdot2\cdot1}\cdot\frac{1}{x^3}\cdot -32x^{15} = \boxed{-1{,}792x^{12}}$$

(b) Find the constant term.

$$\binom{8}{6}\left(\frac{1}{x}\right)^6\left(-2x^3\right)^2 = \frac{8\cdot7}{2\cdot1}\cdot\frac{1}{x^6}\cdot 4x^6 = \boxed{112}$$

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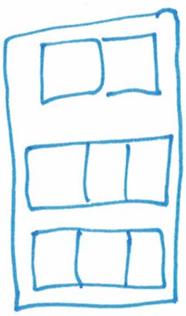
6. A van has eight seats, two at the front, a row of three in the middle, and a row of three at the back. If only 5 out of a group of 8 people can drive, in how many different ways can they be arranged in the car?



$\underline{5\cdot7!} = 25{,}200$

PERMUTATION

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7. A committee of five people is to be selected from a class of $\underbrace{12 \text{ boys and } 9}_{21 \text{ TOTAL}}$ girls. How many such committees include at least 1 girl?

COMBINATION

$$\text{TOTAL} = \binom{21}{5} = \frac{21!}{5!\,16!} = 20{,}349$$

$$\text{NO GIRLS} = \binom{12}{5} = \frac{12!}{5!\,7!} = 792$$

$$\text{AT LEAST 1 GIRL} = 20349 - 792 = \boxed{19{,}557}$$

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8. How many four-digit numbers are there which contain at least one 3?

$\underline{9} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10}$ ALL 4 DIGIT #'S 9,000

$\underline{8} \cdot \underline{9} \cdot \underline{9} \cdot \underline{9}$ NO 3'S $-5,832$

3,168

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9. In the arithmetic series with n^{th} term u_n, it is given that $u_4 = 7$ and $u_9 = 22$.
Find the minimum value of n so that $u_1 + u_2 + u_3 + \ldots + u_n > 10\,000$.

$d = \frac{22-7}{9-4}$

$d = \frac{15}{5}$

$\underline{d = 3}$

$u_4 = u_1 + d(n-1)$

$7 = u_1 + 3(4-1)$

$\underline{u_1 = -2}$

$u_n = -2 + 3(n-1)$

$\underline{u_n = 3n - 5}$

$S_n = \frac{n}{2}(u_1 + u_n)$

$S_n = \frac{n}{2}(-2 + 3n - 5)$

$\underline{S_n = \frac{n}{2}(3n - 7)}$

$\frac{n}{2}(3n-7) > 10,000$

$3n^2 - 7n > 20,000$

$\underline{3n^2 - 7n - 20,000 > 0}$

QUADRATIC FORMULA

$n = \frac{7 \pm \sqrt{49 - 4(3)(-20000)}}{2(3)}$

$n = \frac{7 \pm \sqrt{240,049}}{6}$

$n = \frac{7 \pm 489.9}{6}$

$n = 82.8, -80.5$

$\boxed{n = 83}$
