128. Let $\int_1^5 3f(x)\,dx = 12$.

(a) Show that $\int_5^1 f(x)\,dx = -4$. **(2)**

$\int_1^5 3f(x) = 3\int_1^5 f(x)$

$\frac{3\int_1^5 f(x)}{3} = \frac{12}{3}$

$\int_1^5 f(x) = 4$

$\boxed{\int_5^1 f(x) = -4}$

(b) Find the value of $\int_1^2 (x + f(x))\,dx + \int_2^5 (x + f(x))\,dx$. **(5)**

$\int_1^5 (x+f(x))\,dx = \int_1^5 x \cdot dx + \int_1^5 f(x)\cdot dx = \left[\frac{1}{2}x^2\right]_1^5 + 4 = \left(\frac{25}{2} - \frac{1}{2}\right) + 4$

$\boxed{=16}$

(Total 7 marks)

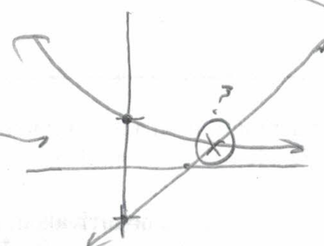
---

129. The curve $y = e^{-x} - x + 1$ intersects the x-axis at P.

(a) Find the x-coordinate of P. **(2)**

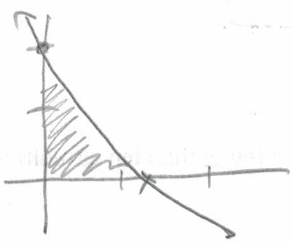
USE GRAPHING CALC

$e^{-x} - x + 1 = 0$ $\quad e^{-x} = x - 1$



$x = 1.2784645$

(b) Find the area of the region completely enclosed by the curve and the coordinate axes. **(3)**



$\int_0^{1.278} (e^{-x} - x + 1)\cdot dx = \left[-e^{-x} - \frac{1}{2}x^2 + x\right]_0^{1.278}$

$= \left(-e^{-1.278} - \frac{1}{2}(1.278)^2 + (1.278)\right) - (-1 + 0 + 0)$

$= -e^{-1.278} - \frac{1}{2}(1.278)^2 + 2.278$

$\approx$

(Total 5 marks)

---

130. Find $\int_0^1 2e^{2x}\,dx$.

$\left[e^{2x}\right]_0^1 = e^2 - e^0 \quad \boxed{= e^2 - 1}$

(Total 3 marks)

131. Over a one month period, Ava and Sven play a total of n games of tennis. The probability that Ava wins any game is 0.4. The result of each game played is independent of any other game played. Let X denote the number of games won by Ava over a one month period.

(a) Find an expression for $P(X = 2)$ in terms of n. **(3)**

$\binom{n}{2}(0.4)^2(0.6)^{n-2}$

$\frac{n!}{2!(n-2)!}(0.4)^2(0.6)^{n-2}$

GUESS + TEST

USE CALC BINOMIAL PDF

$n=10$, $P(x=2) = 0.1209\ldots$

SKIP!!

(b) If the probability that Ava wins two games is 0.121 correct to three decimal places, find the value of n. **(3)**

$\boxed{n=10}$

(Total 6 marks)

---

132. Casualties arrive at an accident unit with a mean rate of one every 10 minutes. Assume that the number of arrivals can be modeled by a Poisson distribution. $X \sim Po(1)$ 10 MIN

(a) Find the probability that there are no arrivals in a given half hour period. **(3)**

FORMULA PACKET $P(x) = \frac{m^x e^{-m}}{x!}$

$X \sim Po(3)$ $P(X=0) = \frac{3^0 e^{-3}}{0!} = \frac{1}{e^3} = \boxed{0.0498}$

$P(x=0)$

(b) A nurse works for a two hour period. Find the probability that there are fewer than ten casualties during this period. **(3)**

$X \sim Po(12)$

$P(x<10) \Rightarrow$ POISSON CDF() $= \boxed{0.242}$

(CALCULATOR)

(Total 6 marks)

---

133. When a boy plays a game at a fair, the probability that he wins a prize is 0.25. He plays the game 10 times. Let X denote the total number of prizes that he wins. Assuming that the games are independent, find

(a) $E(X)$. $E(x) = 10 \cdot 0.25 = \boxed{2.5}$

(b) $P(X \le 2)$. $P(x=0) + P(x=1) + P(x=2)$

$\binom{10}{0}(0.25)^0(0.75)^{10} + \binom{10}{1}(0.25)^1(0.75)^9 + \binom{10}{2}(0.25)^2(0.75)^8$

$0.75^{10} + 10(0.25)(0.75)^9 + 45(0.25)^2(0.75)^8$

(Total 6 marks)

$= \boxed{0.526}$

134. The random variable X has a Poisson distribution with mean 4. Calculate

CALCULATOR

(a) $P(3 \leq X \leq 5)$; $\quad \text{PoissonCDF}(\overset{\lambda}{4}, \overset{L}{3}, \overset{U}{5}) = \boxed{0.547}$ **(2)**

(b) $P(X \geq 3)$; $\quad 1 - P(X<3) = 1 - 0.238 = \boxed{0.762}$ **(2)**

(c) $P(3 \leq X \leq 5 | X \geq 3)$. $= \dfrac{P(3\leq X \leq 5 \cap X \geq 3)}{P(X \geq 3)} = \dfrac{P(3 \leq X \leq 5)}{P(X \geq 3)} = \dfrac{0.547}{0.762} = \boxed{0.718}$ **(2)**

(Total 6 marks)

---

135. Patients arrive at random at an emergency room in a hospital at the rate of 15 per hour throughout the day. Find the probability that 6 patients will arrive at the emergency room between 08:00 and 08:15.

Poisson

$X \sim Po(15)$ per hour

8–8:15 $\quad X \sim Po(3.75)$

$$P(X=6) = \frac{3.75^6 \cdot e^{-3.75}}{6!} = \boxed{0.0908}$$

(Total 3 marks)

---

136. A biased coin is weighted such that the probability of obtaining a head is $\frac{4}{7}$. The coin is tossed 6 times and X denotes the number of heads observed. Find the value of the ratio $\frac{P(X=3)}{P(X=2)}$.

$X \sim B(6, \frac{4}{7})$

$$\frac{P(X=3) = \binom{6}{3}\left(\frac{4}{7}\right)^3\left(\frac{3}{7}\right)^3}{P(X=2) = \binom{6}{2}\left(\frac{4}{7}\right)^2\left(\frac{3}{7}\right)^4} = \frac{\overset{20}{\frac{6!}{3!3!}} \cdot \frac{4}{7}}{\underset{15}{\frac{6!}{2!4!}} \cdot \frac{3}{7}} = \frac{20 \cdot 4}{15 \cdot 3} = \frac{80}{45} = \boxed{\frac{16}{9}}$$

(Total 4 marks)

---

137. X is a binomial random variable, where the number of trials is 5 and the probability of success of each trial is p. Find the values of p if $P(X = 4) = 0.12$.

$X \sim B(5, p)$ $\quad 0 \leq p \leq 1$

$\binom{5}{4}(p)^4(1-p)^1 = 0.12$

$5p^4(1-p) = 0.12$

$5p^4 - 5p^5 - 0.12 = 0$

$5p^5 - 5p^4 + 0.12 = 0$

use calculator (x int)

$\boxed{p = 0.459,\ 0.973}$

(Total 3 marks)

138. A coin is biased so that when it is tossed the probability of obtaining heads is $\frac{2}{3}$. The coin is tossed 1800 times. Let X be the number of heads obtained. Find

(a) the mean of X;

$1800\left(\frac{2}{3}\right) = \boxed{1200}$

(b) the standard deviation of X.

$\sqrt{1800\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)} = \sqrt{400} = \boxed{20}$

(Total 3 marks)

139. In an experiment, a trial is repeated n times. The trials are independent and the probability p of success in each trial is constant. Let X be the number of successes in the n trials. The mean of X is 0.4 and the standard deviation is 0.6.

(a) Find p.

MEAN = np

$\sigma = \sqrt{npq}$

$0.6 = \sqrt{0.4q}$

$0.4q = 0.36$

$q = 0.9$ $\boxed{p = 0.1}$

(b) Find n.

$0.4 = n \cdot 0.1$

$\boxed{n = 4}$

(Total 6 marks)

140. A coin is biased so that when it is tossed the probability of obtaining heads is $\frac{2}{3}$. The coin is tossed 30 times. Let X be the number of heads obtained. Find

$n = 30$ $p = \frac{2}{3}$

(a) the probability of obtaining exactly 10 heads;

$\binom{30}{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{20} = \boxed{0.0001494 \text{ or } 0.015\%}$

16–30

(b) the probability of obtaining more heads than tails.

BINOMIALCDF$\left(30, \frac{2}{3}\right)$ $\boxed{= 0.9565}$

(Total 3 marks)

141. Evaluate, if possible, the following limits. Use ∞ & $-\infty$ as necessary and show all work.

$$\lim_{x\to -1} \begin{Bmatrix} \dfrac{x^2-3x-4}{x+1} & x \neq -1 \\ 2 & x = -1 \end{Bmatrix}$$

$$\lim_{x\to\infty} \frac{x^2}{e^x}$$

$$\lim_{h\to 0} \frac{(x+h)^3-(x^3)}{h} \text{ (in terms of } x\text{)}$$

$$\lim_{h\to 0} \frac{\cos\left(\frac{\pi}{2}+h\right)}{h}$$

PAPER 3

142. ***Multiple Choice. Show work for full credit.***

Which of the following is/are true about the function g if $g(x) = \dfrac{(x-2)^2}{x^2+x-6}$?

I. g is continuous at $x = 2$

II. The graph of g has a vertical asymptote at $x = -3$

III. The graph of g has a horizontal asymptote at $y = 0$

a.) I only b.) II only c.) III only d.) I and II only e.) II and III only

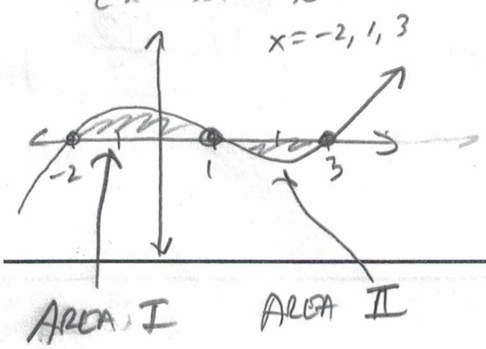
143. Find the area of the region bounded by the graph of $y = \frac{1}{2}(x^3 - 2x^2 - 5x + 6)$ and the x-axis.(Synthetic division)

(1)	1	−2	−5	6
	↓	1	−1	−6
	1	−1	−6	0

$x^2 - 1x - 6$

$(x-3)(x+2)(x-1)$

$x = -2, 1, 3$



IGNORE

AREA I AREA II

$$\frac{1}{2}\int_{-2}^{1}(x^3-2x^2-5x+6)\,dx + \left|\frac{1}{2}\int_{1}^{3}(x^3-2x^2-5x+6)\,dx\right|$$

$$\frac{1}{2}\left[\frac{1}{4}x^4-\frac{2}{3}x^3-\frac{5}{2}x^2+6x\right]_{-2}^{1} + \left|\frac{1}{2}\left[\frac{1}{4}x^4-\frac{2}{3}x^3-\frac{5}{2}x^2+6x\right]_{1}^{3}\right|$$

$$\frac{1}{2}\left(\frac{37}{12}-\left(-\frac{38}{3}\right)\right) + \left|\frac{1}{2}\left(-\frac{9}{4}-\frac{37}{12}\right)\right|$$

$$\frac{63}{8} + \frac{8}{3} = \boxed{\frac{253}{24} \text{ or } 10.541\overline{6}}$$

144. Find the area of the region bounded by the graph of $y = x^3 + 1$, the y-axis, and the lines $y = 1$ and $y = 9$.

$y = x^3 + 1$

$x^3 = y - 1$

$x = \sqrt[3]{y-1}$

$$\int_1^9 (y-1)^{1/3}\,dy = \left[\tfrac{3}{4}(y-1)^{4/3}\right]_1^9$$

$$= \tfrac{3}{4}(16 - 0)$$

$$\boxed{A = 12}$$

or $9 \cdot 2 = 18$

$$18 - \int_0^2 (x^3+1)\,dx$$

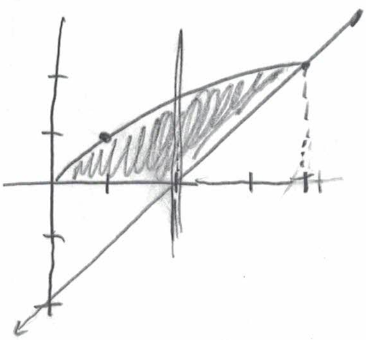
$$18 - \left[\tfrac{1}{4}x^4 + x\right]_0^2$$

$$18 - ((4+2) - 0)$$

$$18 - 6$$

$$\boxed{12}$$

145. Find the area of the region in the first quadrant that is enclosed by $y = \sqrt{x}$, the x-axis, and the line $y = x - 2$.



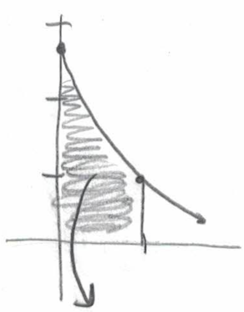
$$A = \int_0^2 \sqrt{x}\,dx + \left(\int_2^4 (\sqrt{x} - (x-2))\,dx\right)$$

$$= \left[\tfrac{2}{3}x^{3/2}\right]_0^2 + \left[\tfrac{2}{3}x^{3/2} - \tfrac{1}{2}x^2 + 2x\right]_2^4$$

$$= \left(\tfrac{4}{3}\sqrt{2} - 0\right) + \left(\tfrac{16}{3} - 3.8856\right)$$

$$\boxed{= \tfrac{10}{3}}$$

146. Find the volume of the solid formed when the graph of the curve $y = e^{1-x}$ is rotated 2π radians about the x-axis between $x = 0$ and $x = 1$.



$$V = \pi \int_0^1 (e^{1-x})^2\,dx$$

$$= \pi \int_0^1 e^{2-2x}\,dx$$

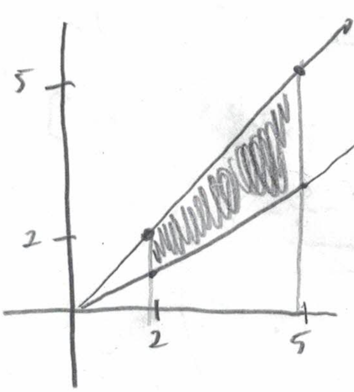
$$= \pi\left[-\tfrac{1}{2}e^{2-2x}\right]_0^1$$

$$\cancel{= -\tfrac{1}{2}\pi}(e^0 - e^2)$$

$$V = \tfrac{1}{2}\pi(e^2 - 1)$$

$$\boxed{\approx 10.036}$$

147. Find the volume of the solid formed when the region between the graphs of the functions $y = x$ and $y = \dfrac{x}{2}$ is rotated through 2π radians about the x-axis between $x = 2$ and $x = 5$.



$$V = \pi \int_2^5 (x)^2\,dx - \pi \int_2^5 \left(\tfrac{x}{2}\right)^2\,dx$$

$$= \pi \int_2^5 x^2\,dx - \pi \int_2^5 \tfrac{1}{4}x^2\,dx$$

$$= \pi\left[\tfrac{1}{3}x^3\right]_2^5 - \pi\left[\tfrac{1}{12}x^3\right]_2^5$$

$$= \pi\left(\tfrac{125}{3} - \tfrac{8}{3}\right) - \pi\left(\tfrac{125}{12} - \tfrac{8}{12}\right)$$

$$= 39\pi - \tfrac{39}{4}\pi$$

$$\boxed{V = \tfrac{117}{4}\pi}$$

INTEGRATE SEPARATELY FOR VOLUME