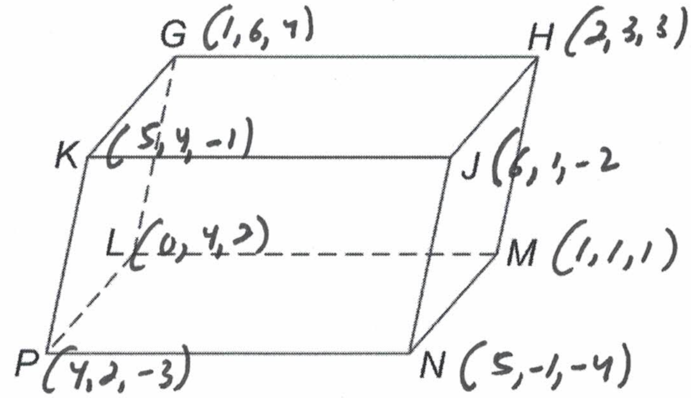
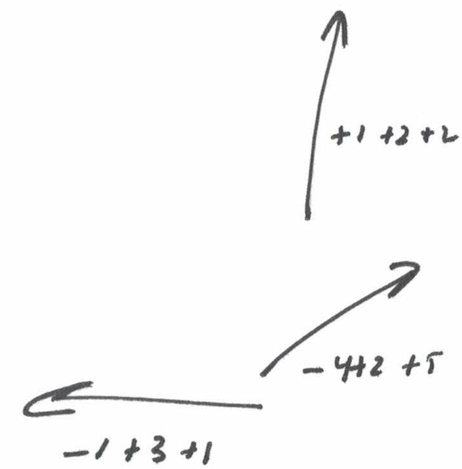
58. The parallelepiped below (all 6 sides are parallelograms) has the following vertices:

$$J(6,1,-2),\ M(1,1,1),\ N(5,-1,-4), \text{ and } P(4,2,-3).$$





a.) Find the coordinates of the remaining 4 points.

(SEE DIAGRAM)

b.) Find $\overrightarrow{PH}$.

$\overrightarrow{PH} = \begin{pmatrix} 2-4 \\ 3-2 \\ 3-(-3) \end{pmatrix} = \boxed{\begin{pmatrix} -2 \\ 1 \\ 6 \end{pmatrix}}$

c.) Find $2\overrightarrow{MN} - \overrightarrow{JG}$.

$-\overrightarrow{JG} = \overrightarrow{GJ}$

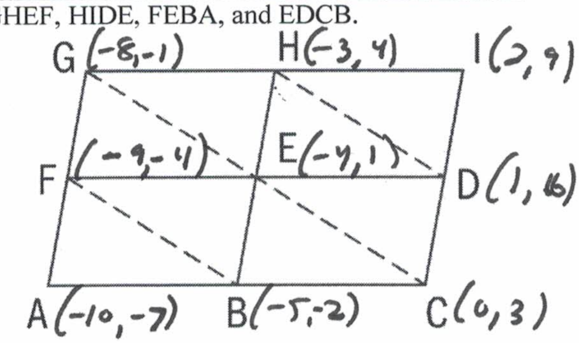
$2\overrightarrow{MN} + \overrightarrow{GJ} = \begin{pmatrix} 8+5 \\ -4-5 \\ -10-6 \end{pmatrix} = \boxed{\begin{pmatrix} 13 \\ -9 \\ -16 \end{pmatrix}}$

$\overrightarrow{MN} = \begin{pmatrix} 4 \\ -2 \\ -5 \end{pmatrix}$

$2 \cdot \overrightarrow{MN} = \begin{pmatrix} 8 \\ -4 \\ -10 \end{pmatrix}$ $\quad \overrightarrow{GJ} = \begin{pmatrix} 5 \\ -5 \\ -6 \end{pmatrix}$

(Total 8 marks)

59. The diagram below is made up of four identical parallelograms GHEF, HIDE, FEBA, and EDCB. Let $G(-8,-1)$, $H(-3,4)$, and $B(-5,-2)$.



a.) Find $\overrightarrow{AI}$.

$\overrightarrow{AI} = \begin{pmatrix} 2-(-10) \\ 9-(-7) \end{pmatrix} = \begin{pmatrix} 12 \\ 16 \end{pmatrix}$

b.) Find the magnitude of $\overrightarrow{CF}$.

$\overrightarrow{CF} = \begin{pmatrix} -9 \\ -7 \end{pmatrix}$ $\quad |\overrightarrow{CF}| = \sqrt{81+49} = \boxed{\sqrt{130}}$

c.) Find the unit vectors collinear to $\overrightarrow{DH}$. +L

$\overrightarrow{DH} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$ $\quad |\overrightarrow{DH}| = \sqrt{16+4} = 2\sqrt{5}$ $\quad u = \pm \frac{1}{2\sqrt{5}} \begin{pmatrix} -4 \\ -2 \end{pmatrix} = \boxed{\pm \frac{1}{\sqrt{5}} \begin{pmatrix} -2 \\ -1 \end{pmatrix}}$

d.) Find a vector with the same magnitude as $\overrightarrow{AH}$ in the same direction as $\overrightarrow{AG}$. Write your answer in the form $\begin{pmatrix} x \\ y \end{pmatrix}$ where $x, y \in \mathbb{R}$.

$AG = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$ $\quad v = \frac{\sqrt{170}}{\sqrt{40}} \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

$\overrightarrow{AH} = \begin{pmatrix} 7 \\ 11 \end{pmatrix}$ $\quad |\overrightarrow{AH}| = \sqrt{49+121} = \sqrt{170}$ $\quad |\overrightarrow{AG}| = \sqrt{4+36} = \sqrt{40}$

(Total 13 marks)

$v = \frac{\sqrt{17}}{2} \begin{pmatrix} 2 \\ 6 \end{pmatrix}$ or $\boxed{\sqrt{17} \begin{pmatrix} 1 \\ 3 \end{pmatrix}}$

60. Given the points $P(6,0,3)$, $Q(2,-4,-1)$, and $R(-1,7,-3)$, find:

(a) A vector equation of line $\overleftrightarrow{PQ}$.

$$\overrightarrow{PQ} = \begin{pmatrix} -4 \\ -4 \\ -4 \end{pmatrix} \quad \boxed{l_1 = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} + a\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}} \leftarrow$$

DIRECTION VECTOR SCALED DOWN BY $\frac{1}{4}$

(b) Parametric equations for line $\overleftrightarrow{QR}$.

$$\overrightarrow{QR} = \begin{pmatrix} -3 \\ 11 \\ -2 \end{pmatrix} \quad l_2 = \begin{pmatrix} 2 \\ -4 \\ -1 \end{pmatrix} + b\begin{pmatrix} -3 \\ 11 \\ -2 \end{pmatrix}$$

$$\boxed{\begin{aligned} x &= 2 - 3b \\ y &= -4 + 11b \\ z &= -1 - 2b \end{aligned}}$$

(c) Cartesian equations for line $\overleftrightarrow{PR}$.

$$\overrightarrow{PR} = \begin{pmatrix} -7 \\ 7 \\ -6 \end{pmatrix} \quad l_3 = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} -7 \\ 7 \\ -6 \end{pmatrix} \quad \begin{aligned} x &= 6 - 7\lambda \\ y &= 0 + 7\lambda \\ z &= 3 - 6\lambda \end{aligned}$$

SOLVE FOR λ

$$\boxed{\frac{x-6}{-7} = \frac{y}{7} = \frac{z-3}{-6}}$$

(d) Any 3 points **other than P, Q, and R** which lie on line $\overleftrightarrow{QR}$.

FROM PART (b)

b	-3	-2	-1	0	1	2	3	4
POINT	$(11,-37,5)$	$(8,-26,3)$	$(5,-15,1)$	$(2,-4,-1)$	$(-1,7,-3)$	$(-4,18,-5)$	$(-7,29,-7)$	$(-10,40,-9)$

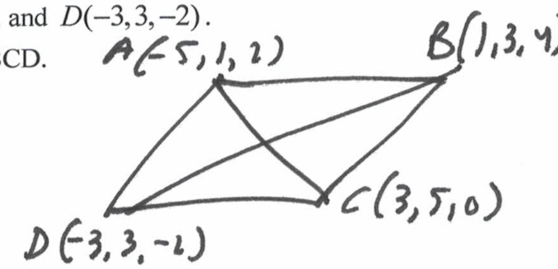
(e) Find the value of $\overrightarrow{PQ} \bullet \overrightarrow{PR}$.

$$\begin{pmatrix} -4 \\ -4 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} -7 \\ 7 \\ -6 \end{pmatrix} = 28 - 28 + 24 = \boxed{24}$$

(Total 11 marks)

---

61. Parallelogram ABCD is given by the coordinates: $A(-5,1,2)$, $B(1,3,4)$, and $D(-3,3,-2)$. Let O represent the origin and P the intersection of the diagonals of ABCD.



(a) Find the coordinates of point C.

$C(3,5,0)$

Let O represent the origin and P the intersection of the diagonals of ABCD.

$(0,0,0)$

(b) Find the magnitude of vector OP.

P MIDPOINT OF $\overline{AC}$ AND $\overline{BD}$

$$\left(\frac{-5+3}{2}, \frac{1+5}{2}, \frac{2+0}{2}\right)$$

$P(-1,3,1)$

$$\overrightarrow{OP} = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} \quad |\overrightarrow{OP}| = \sqrt{1+9+1} = \boxed{\sqrt{11}}$$

(c) Find $\overrightarrow{PB} \bullet \overrightarrow{CP}$.

$$\overrightarrow{PB} \cdot \overrightarrow{CP} = \overrightarrow{PB} \cdot -\overrightarrow{PC} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -2 \\ +1 \end{pmatrix} = -8 + 0 + 3 = \boxed{-5}$$

(d) Are the diagonals of ABCD perpendicular to each other? Briefly explain.

$$\overrightarrow{AC} \cdot \overrightarrow{BD} = \begin{pmatrix} 8 \\ 4 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 0 \\ -6 \end{pmatrix} = -32 + 0 + 12 = -20$$

NO, DOT PRODUCT $\neq 0$

(Total 7 marks)

62. Find the angle formed by the lines $\dfrac{x-3}{2} = 2y = \dfrac{1-z}{7}$ and $x = 2y = z + 5$.

$\theta = \cos^{-1}\left(\left|\dfrac{u \cdot v}{|u||v|}\right|\right)$

$x = 3 + 2a$

$y = 0 + \frac{1}{2}a$

$z = 1 - 7a$

$x = 0 + B$

$y = 0 + \frac{1}{2}B$

$z = -5 + B$

DIRECTION VECTORS

$u = \begin{pmatrix} 4 \\ 1 \\ -14 \end{pmatrix}$ $v = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

$l_1 = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + a\begin{pmatrix} 4 \\ 1 \\ -14 \end{pmatrix}$ $l_2 = \begin{pmatrix} 0 \\ 0 \\ -5 \end{pmatrix} + B\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

SCALED UP x2 SCALED UP x2

$|u| = \sqrt{16+1+196} = \sqrt{213}$

$|v| = \sqrt{4+1+4} = 3$

$u \cdot v = 8 + 1 - 28 = -19$

$\theta = \cos^{-1}\left(\left|\dfrac{-19}{\sqrt{213} \cdot 3}\right|\right)$ $\boxed{\theta = 64.28^\circ}$

---

63. The position vector of point A is $2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ and the position vector of point B is $4\boldsymbol{i} - 5\boldsymbol{j} + 21\boldsymbol{k}$.

(a) Find the unit vector $\boldsymbol{u}$ in the direction of $\overrightarrow{AB}$.

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -8 \\ 20 \end{pmatrix}$ $|\overrightarrow{AB}| = \sqrt{4+64+400} = \sqrt{468} = 6\sqrt{13}$

$\boxed{u = \dfrac{1}{6\sqrt{13}}\begin{pmatrix} 2 \\ -8 \\ 20 \end{pmatrix}}$

$u = \dfrac{1}{\sqrt{13}}\begin{pmatrix} 1/3 \\ -4/3 \\ 10/3 \end{pmatrix}$

(b) Show that $\boldsymbol{u}$ is perpendicular to $\overrightarrow{OA}$.

$\dfrac{1}{\sqrt{13}}\begin{pmatrix} \frac{1}{3} \\ -\frac{4}{3} \\ \frac{10}{3} \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = \dfrac{1}{\sqrt{13}}\left(\dfrac{2}{3} - \dfrac{12}{3} + \dfrac{10}{3}\right) = \boxed{0}$ ✓ PERPENDICULAR

(Total 4 marks)

---

64. The line L_1 is represented by $\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the line L_2 by $\boldsymbol{r}_2 = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$.

The lines L_1 and L_2 intersect at point T. Find the coordinates of T.

r_1 r_2

$x = 2 + 1s$ $x = 3 - 1t$

$y = 5 + 2s$ $y = -3 + 3t$

$z = 3 + 3s$ $z = 8 - 4t$

$x: 2 + 1s = 3 - t$

$s + t = 1$

$s = 1 - t$

$5 + 2s = -3 + 3t$

$2s - 3t = -8$

$2(1-t) - 3t = -8$

$2 - 2t - 3t = -8$

$-5t = -10$

$t = 2$

$t = 2, s = -1$

$x = 1$ $y = 3$ $z = 0$

$\boxed{T(1, 3, 0)}$

(Total 6 marks)

65. A ray of light coming from the point $(-1, 3, 2)$ is travelling in the direction of vector $\begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$ and meets the plane $\pi : x + 3y + 2z - 24 = 0$. Find the angle that the ray of light makes with the plane.

RAY

$l = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} + a\begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$

PLANE

$x + 3y + 2z = 24$

$n = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

$u = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} \quad n = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

$u \cdot n = 4 + 3 - 4 = 3$

$|u| = \sqrt{16+1+4} = \sqrt{21}$

$|n| = \sqrt{1+9+4} = \sqrt{14}$

$\sin\theta = \frac{3}{\sqrt{21}\sqrt{14}}$

$\boxed{\theta = 10.08°}$

(Total 6 marks)

---

66. The vector equation of line l is given as $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$.

Find the Cartesian equation of the plane containing the line l and the point A(4, –2, 5).

$x = 1 - 1\lambda$

$y = 3 + 2\lambda$

$z = 6 - 1\lambda$

$l_1 = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$

$\lambda = 0 \quad B(1, 3, 6)$

$A(4, -2, 5)$

$\vec{BA} = \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix}$

$\begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix} \times \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 5-(-2) \\ 1-(-3) \\ 6-5 \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix}$

$7x + 4y + 1z = d$

$7(4) + 4(-2) + 1(5) = d$

$d = 25$

$\boxed{7x + 4y + 1z = 25}$

(Total 6 marks)

---

67. Given the complex numbers $z_1 = -4 + 4i\sqrt{3}$ and $z_2 = 3\text{ cis}\left(\frac{\pi}{4}\right)$, find the following. Express all answers in modulus-argument form where $r > 0$ and $0 \le \theta < 2\pi$. All angles must be expressed in radians.

a.) $(z_1)\cdot(z_2)$

$8\text{cis}\left(\frac{2\pi}{3}\right) \cdot 3\text{cis}\left(\frac{\pi}{4}\right)$

$24\text{cis}\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

$\boxed{24\text{cis}\left(\frac{11\pi}{12}\right)}$

b.) $(z_1{}^*)\cdot(-z_2)$

$8\text{cis}\left(\frac{4\pi}{3}\right) \cdot -3\text{cis}\left(\frac{\pi}{4}\right)$

$-24\text{cis}\left(\frac{4\pi}{3} + \frac{\pi}{4}\right)$

$-24\text{cis}\left(\frac{19\pi}{12}\right)$

$\boxed{24\text{cis}\left(\frac{7\pi}{12}\right)}$

c.) $\left(\frac{z_1}{z_2}\right)^*$

$\left(\frac{8\text{cis}\left(\frac{2\pi}{3}\right)}{3\text{cis}\left(\frac{\pi}{4}\right)}\right)^*$

$\left(\frac{8}{3}\text{cis}\left(\frac{2\pi}{3} - \frac{\pi}{4}\right)\right)^*$

$\left(\frac{8}{3}\text{cis}\left(\frac{5\pi}{12}\right)\right)^*$

$\frac{8}{3}\text{cis}\left(-\frac{5\pi}{12}\right)$

$\boxed{\frac{8}{3}\text{cis}\left(\frac{19\pi}{12}\right)}$

68. Given that $\frac{z}{z+2} = 2 - \mathrm{i}$, $z \in \mathbb{C}$, find z in the form $a + \mathrm{i}b$.

SKIP... REPEAT OF #53

(Total 4 marks)

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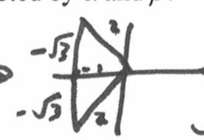
69. The roots of the equation $z^2 + 2z + 4 = 0$ are denoted by α and β?

(a) Find α and β in the form $r\mathrm{e}^{\mathrm{i}\theta}$. **(6)**

$$z = \frac{-2 \pm \sqrt{4 - 4(1)(4)}}{2}$$

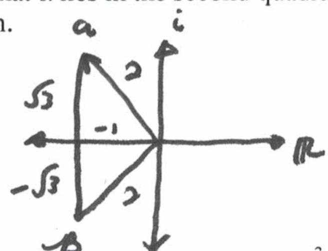
$$z = -1 \pm \sqrt{3}$$



$\alpha = 2\,cis\left(\frac{2\pi}{3}\right)$ or $2e^{i\cdot\frac{2\pi}{3}}$

$\beta = 2\,cis\left(\frac{4\pi}{3}\right)$ or $2e^{i\cdot\frac{4\pi}{3}}$

(b) Given that α lies in the second quadrant of the Argand diagram, mark α and β on an Argand diagram. **(2)**



(c) Using De Moivre's theorem find $\frac{\alpha^3}{\beta^2}$ in the form $a + \mathrm{i}b$. **(4)**

$$\frac{\left[2\,cis\left(\frac{2\pi}{3}\right)\right]^3}{\left[2\,cis\left(\frac{4\pi}{3}\right)\right]^2} = \frac{8\,cis\left(3\cdot\frac{2\pi}{3}\right)}{4\,cis\left(2\cdot\frac{4\pi}{3}\right)} = 2\,cis\left(2\pi - \frac{8\pi}{3}\right) = 2\,cis\left(-\frac{2\pi}{3}\right) = 2\,cis\left(\frac{4\pi}{3}\right) = \beta$$

$$\boxed{-1 - \sqrt{3}}$$

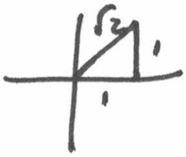
(d) Using De Moivre's theorem or otherwise, show that $\alpha^3 = \beta^3$. **(3)**

$\alpha^3 = 8\,cis(2\pi) = 8\,cis\,0 = 8$ (SEE ABOVE)

$\beta^3 = \left[2\,cis\left(\frac{4\pi}{3}\right)\right]^3 = 8\,cis\left(3\cdot\frac{4\pi}{3}\right) = 8\,cis(4\pi) = 8\,cis\,0 = 8$

(Total 15 marks)

70. (a) Express the complex number $1+\mathrm{i}$ in the form $\sqrt{a}e^{i\frac{\pi}{b}}$, where $a, b \in \mathbb{Z}^+$. **(2)**



$= \sqrt{2}\,\text{cis}\left(\frac{\pi}{4}\right) = \sqrt{2}e^{i\cdot\frac{\pi}{4}}$

(b) Using the result from (a), show that $\left(\frac{1+\mathrm{i}}{\sqrt{2}}\right)^n$, where $n \in \mathbb{Z}$, has only eight distinct values. **(5)**

$\left(\frac{1+i}{\sqrt{2}}\right)^n = \frac{\sqrt{2}e^{i\cdot\frac{\pi}{4}}}{\sqrt{2}} = \left(e^{i\cdot\frac{\pi}{4}}\right)^n = 1^n e^{i\cdot\frac{\pi}{4}\cdot n} = e^{in\cdot\frac{\pi}{4}}$

$n\cdot\frac{\pi}{4} = \boxed{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4},}\ \underset{=0}{2\pi} \ldots$ REPEATS

(c) **Hence** solve the equation $z^8 - 1 = 0$. **(2)**

$z^8 = 1$

$z^8 = 1 + 0i$

$z^8 = 1\,\text{cis}\,0$

$\sqrt[8]{1}\,\text{cis}\left(\frac{0+2\pi k}{8}\right)$ $k = 0, 1, 2, \ldots, 7$

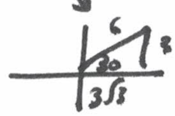
$z = 1\,\text{cis}\left(0 + \frac{\pi}{4}\cdot k\right)$

$z = 1\,\text{cis}\,(n)$, $n = 0, \frac{\pi}{4}, \ldots, \frac{7\pi}{4}$

(Total 9 marks)

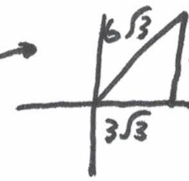
71. Find the following sum, expressing your answer in modulus-argument form.

$$6cis\left(\frac{\pi}{2}\right) + 6cis\left(\frac{\pi}{6}\right)$$



$z_1 = 0 + 6i$ $z_2 = 3\sqrt{3} + 3i$

$z_1 + z_2 = 3\sqrt{3} + 9i$



$3\sqrt{3}\cdot\sqrt{3} = 9$

30 - 60 - 90

or

$(3\sqrt{3})^2 + 9^2 = c^2$

$27 + 81 = c^2$

$c = \sqrt{108}$ or $6\sqrt{3}$

$\boxed{6\sqrt{3}\,\text{cis}\left(\frac{\pi}{3}\right)}$

(Total 3 marks)

72. Consider the complex number $\omega = \frac{z+\mathrm{i}}{z+2}$, where $z = x + \mathrm{i}y$. If $\omega = \mathrm{i}$, determine z in the form $z = r$ cis θ.

$i = \frac{z+i}{z+2}$

$i(z+2) = z+i$

$iz + 2i = z + i$

$iz - z = -i$

$z(-1+i) = -i$

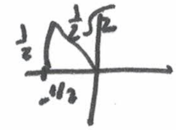
$z = \frac{-i}{-1+i}$

$z = \frac{-i(-1-i)}{-1+i(-1-i)}$

$z = \frac{-1+i}{1-i^2}$

$z = \frac{-1+i}{2}$

$z = -\frac{1}{2} + \frac{1}{2}i$



$\boxed{z = \frac{\sqrt{2}}{2}\,\text{cis}\left(\frac{3\pi}{4}\right)}$

(Total 6 marks)

73. The complex number z is defined by

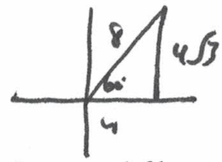
$-c+s$

$$z = 4\left(\cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}\right) + 4\sqrt{3}\left(\cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}\right).$$

(a) Express z in the form $re^{\mathrm{i}\theta}$, where r and θ have exact values.

$z = -2 + 2\sqrt{3}i + 6 + 2\sqrt{3}i$

$z = 4 + 4\sqrt{3}i$



$z = 8 cis\left(\frac{\pi}{3}\right)$

$z = 8e^{i\cdot\frac{\pi}{3}}$ $\quad r = 8 \quad \theta = \frac{\pi}{3}$

(b) Find the cube roots of z, expressing in the form $re^{\mathrm{i}\theta}$, where r and θ have exact values.

$\sqrt[3]{z} = \sqrt[3]{8 cis\left(\frac{\pi}{3}\right)} = \sqrt[3]{8}\, cis\left(\frac{\frac{\pi}{3} + 2\pi n}{3}\right) \quad n = 0, 1, 2$

$= 2 cis\left(\frac{\pi}{9} + \frac{2\pi}{3}\cdot n\right)$

$\sqrt[3]{z} = 2 cis\frac{\pi}{9}$

$2 cis\left(\frac{7\pi}{9}\right)$

$2 cis\left(\frac{13\pi}{9}\right)$

(Total 6 marks)

---

74. (a) Use de Moivre's theorem to find the roots of the equation $z^4 = 1 - \mathrm{i}$. **(6)**

$z^4 = \sqrt{2} cis\left(\frac{7\pi}{4}\right)$ $\qquad z = \sqrt[8]{2}\, cis\left(\frac{\frac{7\pi}{4} + 2\pi\cdot n}{4}\right)$

$z = \sqrt[4]{\sqrt{2} cis\left(\frac{7\pi}{4}\right)}$ $\qquad z = \sqrt[8]{2} cis\left(\frac{7\pi}{16} + \frac{\pi}{2}\cdot n\right) \quad n = 0, 1, 2, 3$

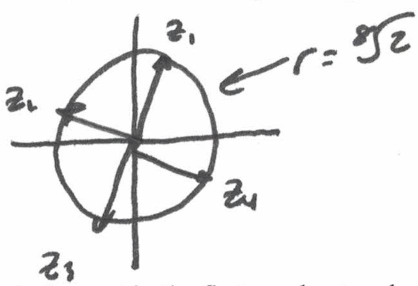
$z_1 = \sqrt[8]{2} cis\left(\frac{7\pi}{16}\right)$

$z_2 = \sqrt[8]{2} cis\left(\frac{15\pi}{16}\right)$

$z_3 = \sqrt[8]{2} cis\left(\frac{23\pi}{16}\right)$

$z_4 = \sqrt[8]{2} cis\left(\frac{31\pi}{16}\right)$

(b) Draw these roots on an Argand diagram. **(2)**



(b) If z_1 is the root in the first quadrant and z_2 is the root in the second quadrant, find $\frac{z_2}{z_1}$ in the form $a + \mathrm{i}b$. **(4)**

$\frac{z_2}{z_1} = \frac{\sqrt[8]{2} cis\left(\frac{15\pi}{16}\right)}{\sqrt[8]{2} cis\left(\frac{7\pi}{16}\right)} = 1 cis\left(\frac{15\pi}{16} - \frac{7\pi}{16}\right) = 1 cis\left(\frac{\pi}{2}\right)$ or i

(Total 12 marks)

---

75. Let $f(x) = x^3 - 2x^2 - 1$.

(a) Find $f'(x)$.

(b) Find the gradient of the curve of $f(x)$ at the point $(2, -1)$.

(a) $\boxed{f'(x) = 3x^2 - 4x}$

(b) $f'(2) = 3(2)^2 - 4(2)$ $\quad \boxed{f'(2) = 4}$

(Total 6 marks)

76. A gradient function is given by $\frac{dy}{dx} = 10e^{2x} - 5$. When $x = 0$, $y = 8$. Find the value of y when $x = 1$.

$y = 5e^{2x} - 5x + c$

$8 = 5e^{0} - 5(0) + c$

$c = 3$

$y = 5e^{2x} - 5x + 3$

$y = 5e^{2} - 5(1) + 3$

$\boxed{y = 5e^{2} - 2}$

(Total 8 marks)

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77. For what values of m is the line $y = mx + 5$ a tangent to the parabola $y = 4 - x^2$?

$y = 4 - x^2$

$\frac{dy}{dx} = -2x$

$m = -2x$

$y = (-2x)(x) + 5$

$y = -2x^2 + 5$

$-2x^2 + 5 = 4 - x^2$

$x^2 = 1$

$x = \pm 1$

$\boxed{m = \pm 2}$

(Total 3 marks)

---

78. Consider the function $f(x) = x^3 - 3x^2 - 9x + 10$, $x \in \mathbb{R}$.

(a) Find the equation of the straight line passing through the maximum and minimum points of the graph $y = f(x)$. **(4)**

$f'(x) = 3x^2 - 6x - 9$

$3(x^2 - 2x - 3) = 0$

$3(x-3)(x+1) = 0$

$x = 3, -1$

$f(3) = -17$

$f(-1) = 15$

$m = \frac{15 - (-17)}{-1 - 3}$

$m = -8$

$-17 = -8(3) + b$

$b = 7$

$\boxed{y = -8x + 7}$

(b) Show that the point of inflexion of the graph $y = f(x)$ lies on this straight line. **(2)**

$f''(x) = 6x - 6$

$6x - 6 = 0$

$x = 1$

$f(1) = (1)^3 - 3(1)^2 - 9(1) + 10$

$\boxed{f(1) = -1}$

$y = -8(1) + 7$

$\boxed{y = -1}$

(Total 3 marks)

---

79. The function f is given by $f(x) = \frac{x^5 + 2}{x}$, $x \neq 0$. There is a point of inflexion on the graph of f at the point P. Find the coordinates of P.

MAY USE QUOTIENT RULE

$f(x) = x^4 + 2x^{-1}$

$f'(x) = 4x^3 - 2x^{-2}$

$f''(x) = 12x^2 + 4x^{-3}$

$12x^2 + 4x^{-3} = 0$

$4x^{-3}(3x^5 + 1) = 0$

$x^5 = \frac{-1}{3}$

$x = \frac{-1}{\sqrt[5]{3}}$ or $-3^{-1/5}$

$f(-3^{-1/5}) = (-3^{-1/5})^4 + 2(-3^{-1/5})^{-1}$

$= 3^{-4/5} - 2 \cdot 3^{1/5}$

$= -5 \cdot 3^{-4/5}$

$\boxed{P(-3^{-1/5}, -5 \cdot 3^{-4/5})}$

(Total 6 marks)

80. The displacement s meters of a moving body B from a fixed point O at time t seconds is given by

$$s = 50t - 10t^2 + 1000.$$

(a) Find the velocity of B in m s^{-1}.

(b) Find its maximum displacement from O.

(a) $\frac{ds}{dt} = v = 50 - 20t$

(b) $50 - 20t = 0$

$t = 2.5$

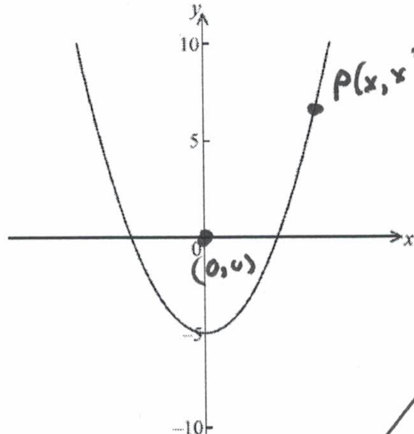
$s(2.5) = 50(2.5) - 10(2.5)^2 + 1000 = 1062.5$

$s(0) = 1000$

$\boxed{62.5}$ (AT 1000 $t=0$, AT 1062.5 $t=2.5$)

(Total 6 marks)

81. The curve $y = x^2 - 5$ is shown below, with point O at the origin.



(a) $D = \sqrt{(x^2-5-0)^2 + (x-0)^2}$

$= \sqrt{x^4 - 10x^2 + 25 + x^2}$

$= \boxed{\sqrt{x^4 - 9x^2 + 25}}$

$x = a$

A point P on the curve has x-coordinate equal to a.

(a) Show that the distance OP is $\sqrt{a^4 - 9a^2 + 25}$.

(b) $D(x) = (x^4 - 9x^2 + 25)^{1/2}$

$D'(x) = \frac{1}{2}(x^4 - 9x^2 + 25)^{-1/2} \cdot (4x^3 - 18x)$

(2)

$4x^3 - 18x = 0$

$2x(2x^2 - 9) = 0$

$x = 0, \pm\sqrt{\frac{9}{2}}$ or $\pm\frac{3}{\sqrt{2}}$

(b) Find the values of ~~a~~ x for which the curve is closest to the origin. **(5)**

$-$ $+$ $-$ $+$

$-\sqrt{\frac{9}{2}}$ 0 $\sqrt{\frac{9}{2}}$

MIN MAX MIN

$a = \pm\sqrt{\frac{9}{2}}$

(Total 7 marks)

82. Find the equation of the normal to the curve $5xy^2 - 2x^2 = 18$ at the point (1, 2).

$5y^2 + 5x \cdot 2y \cdot \frac{dy}{dx} - 4x = 0$

$5(2)^2 + 10(1)(2) \cdot \frac{dy}{dx} - 4(1) = 0$

$20 \cdot \frac{dy}{dx} = -16$

$\frac{dy}{dx} = -\frac{4}{5}$ TANGENT

$m = \frac{5}{4}$ NORMAL

$y = mx + b$

$2 = \frac{5}{4}(1) + b$

$b = \frac{3}{4}$

$\boxed{y = \frac{5}{4}x + \frac{3}{4}}$

(Total 7 marks)

83. Find the equation of the normal to the curve $3x^2y + 2xy^2 = 2$ at the point $(1, -2)$.

$6xy + 3x^2\cdot\frac{dy}{dx} + 2y^2 + 2x\cdot 2y\cdot\frac{dy}{dx} = 0$

$6xy + 3x^2\cdot\frac{dy}{dx} + 2y^2 + 4xy\cdot\frac{dy}{dx} = 0$

$6(1)(-2) + 3(1)^2\cdot\frac{dy}{dx} + 2(-2)^2 + 4(1)(-2)\cdot\frac{dy}{dx} = 0$

$-12 + 3\cdot\frac{dy}{dx} + 8 - 8\cdot\frac{dy}{dx} = 0$

$-5\cdot\frac{dy}{dx} = 4$

$\frac{dy}{dx} = -\frac{4}{5}$ TANG.

$m = \frac{5}{4}$ NORMAL

$y = mx + b$

$-2 = \frac{5}{4}(1) + b$

$b = -\frac{13}{4}$

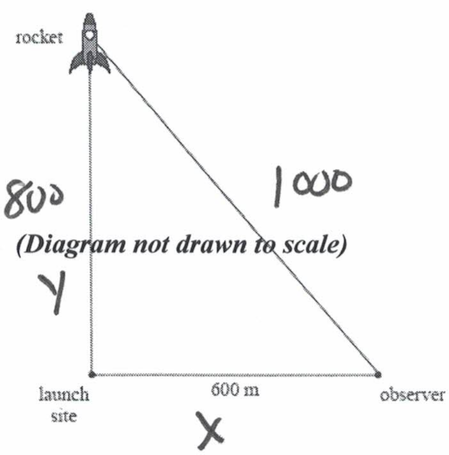
$\boxed{y = \frac{5}{4}x - \frac{13}{4}}$

(Total 7 marks)

---

84. A rocket is rising vertically at a speed of 300 m s^{-1} when it is 800 m directly above the launch site. Calculate the rate of change of the distance between the rocket and an observer, who is 600 m from the launch site and on the same horizontal level as the launch site.

$\frac{dy}{dx} = 300$



(Diagram not drawn to scale)

$600^2 + y^2 = d^2$

$\cancel{0} + 2y\cdot\frac{dy}{dt} = 2d\cdot\frac{dd}{dt}$

$2(800)(300) = 2(1000)\cdot\frac{dd}{dt}$

$\boxed{\frac{dd}{dt} = 240 \text{ m/s}}$

(Total 6 marks)

---

85. If $f(x) = x - 3x^{\frac{2}{3}}$, $x > 0$,

(a) find the x-coordinate of the point P where $f'(x) = 0$; **(2)**

$f'(x) = 1 - 2x^{-1/3}$

$1 - 2x^{-1/3} = 0$

$2x^{-1/3} = 1$

$x^{-1/3} = \frac{1}{2}$

$(x^{1/3} = 2)$ CUBE BOTH SIDES

$\boxed{x = 8}$

(b) determine and show whether P is a maximum or minimum point. **(3)**

$-$ $+$

1 8 10

(-1) (0.07)

$\boxed{\text{MINIMUM}}$

(Total 5 marks)

86. Find the gradient of the curve $e^{xy} + \ln(y^2) + e^y = 1 + e$ at the point (0, 1).

CONSTANT

$$e^{xy}\cdot\underbrace{\left(y + x\cdot\frac{dy}{dx}\right)}_{\text{CHAIN RULE}} + \frac{1}{y^2}\cdot 2y\cdot\frac{dy}{dx} + e^y\cdot\frac{dy}{dx} = 0$$

$$@(0,1)\quad e^0(1+0) + \left(1\cdot 2\cdot\frac{dy}{dx}\right) + \left(e^1\cdot\frac{dy}{dx}\right) = 0$$

$$1 + 2\cdot\frac{dy}{dx} + e\cdot\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2+e) = -1$$

$$\boxed{\frac{dy}{dx} = \frac{-1}{2+e}}$$

(Total 7 marks)

---

87. The curve $y = e^{-x} - x + 1$ intersects the x-axis at P.

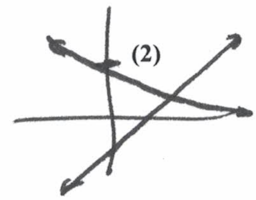
(a) Find the x-coordinate of P. **(2)**

MUST USE A GRAPHING CALCULATOR!

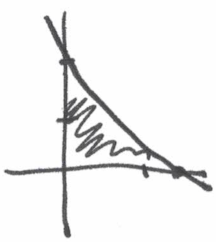
$$e^{-x} - x + 1 = 0$$

$$e^{-x} = x - 1$$

$$x = 1.2784645$$



(b) Find the area of the region completely enclosed by the curve and the coordinate axes. **(3)**



$$A = \int_0^{1.278} e^{-x} - x + 1 \cdot dy = \left[-e^{-x} - \tfrac{1}{2}x^2 + x\right]_0^{1.278}$$

$$= \left(-e^{-1.278} - \tfrac{1}{2}(1.278)^2 + (1.278)\right) - (-1 + 0 + 0)$$

$$= -e^{-1.278} - \tfrac{1}{2}(1.278)^2 + 2.278$$

$$\boxed{A = 1.18327427645}$$

(Total 5 marks)

---

88. Over a one month period, Ava and Sven play a total of n games of tennis. The probability that Ava wins any game is 0.4. The result of each game played is independent of any other game played. Let X denote the number of games won by Ava over a one month period.

(a) Find an expression for $P(X = 2)$ in terms of n. **(3)**

$$\binom{n}{2}(0.4)^2(0.6)^{n-2}$$

$$\frac{n!}{2!(n-2)!}(0.4)^2(0.6)^{n-2}$$

GUESS + CHECK USING BINOMIAL PDF ON TI-NSPIRES. NO GOOD ALGEBRAIC METHOD.

$n = 10,\ P(X=2) = 0.1209\ldots$

(b) If the probability that Ava wins two games is 0.121 correct to three decimal places, find the value of n. **(3)**

$n = 10$

(Total 6 marks)

89. A biased coin is weighted such that the probability of obtaining a head is $\frac{4}{7}$. The coin is tossed 6 times and X denotes the number of heads observed. Find the value of the ratio $\frac{P(X=3)}{P(X=2)}$.

$X \sim B(6, \frac{4}{7})$

$$\frac{P(X=3)}{P(X=2)} = \frac{\binom{6}{3}\left(\frac{4}{7}\right)^3\left(\frac{3}{7}\right)^3}{\binom{6}{2}\left(\frac{4}{7}\right)^2\left(\frac{3}{7}\right)^4} = \frac{\overset{20}{\frac{6!}{3!\,3!}}\cdot\frac{4}{7}}{\underset{15}{\frac{6!}{2!\,4!}}\cdot\frac{3}{7}} = \frac{20\cdot 4}{15\cdot 3} = \frac{80}{45} = \boxed{\frac{16}{9}}$$

(Total 4 marks)

---

90. A coin is biased so that when it is tossed the probability of obtaining heads is $\frac{2}{3}$. The coin is tossed 1800 times. Let X be the number of heads obtained. Find

(a) the mean, also known as the expected value E(X), of X;

$$E(x) = np = 1800\left(\tfrac{2}{3}\right) = \boxed{1200}$$

(b) the standard deviation of X.

$$\text{STDEV} = \sqrt{npq} = \sqrt{1800\left(\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right)} = \sqrt{400} = \boxed{20}$$

(Total 3 marks)

---

91. In an experiment, a trial is repeated n times. The trials are independent and the probability p of success in each trial is constant. Let X be the number of successes in the n trials. The mean of X is 0.4 and the standard deviation is 0.6.

(a) Find p.

$\mu = np$

$\sigma = \sqrt{npq}$

$0.6 = \sqrt{0.4\cdot q}$

$0.4q = 0.36$

$q = 0.9$

$\boxed{p = 0.1}$

(b) Find n.

$0.4 = n \cdot 0.1$

$\boxed{n = 4}$

(Total 6 marks)

---

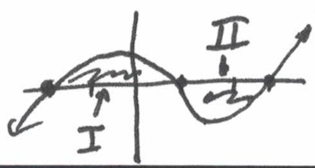
92. Find the area of the region bounded by the graph of $y = \frac{1}{2}(x^3 - 2x^2 - 5x + 6)$ and the x-axis.(Synthetic division)

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 6 \\ & \downarrow & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

$x^2 - x - 6$

$(x-1)(x-3)(y+2)$

$x = -2, 1, 3$



AREA I

$A = \frac{1}{2}\int_{-2}^{1} x^3 - 2x^2 - 5x + 6 \, dy$

CALCULATOR

$A = \frac{63}{8}$

AREA II

$A = \left| \frac{1}{2}\int_{1}^{3} x^3 - 2x^2 - 5x + 6 \, dy \right|$

$A = \left| -\frac{8}{3} \right|$

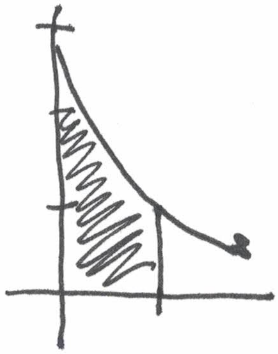
$A = \frac{8}{3}$

TOTAL AREA $= \frac{63}{8} + \frac{8}{3} = \boxed{\frac{253}{24} \text{ or } 10.541\bar{6}}$

93. Find the area of the region bounded by the graph of $y = x^3 + 1$, the y-axis, and the lines $y = 1$ and $y = 9$.

94. Find the area of the region in the first quadrant that is enclosed by $y = \sqrt{x}$, the x-axis, and the line $y = x - 2$.

95. Find the volume of the solid formed when the graph of the curve $y = e^{1-x}$ is rotated 2π radians about the x-axis between $x = 0$ and $x = 1$.



$$V = \pi \int_0^1 \left(e^{1-x}\right)^2 \cdot dx$$

$$= \pi \int_0^1 e^{2-2x} \cdot dx$$

$$= \pi \left[-\tfrac{1}{2} e^{2-2x}\right]_0^1$$

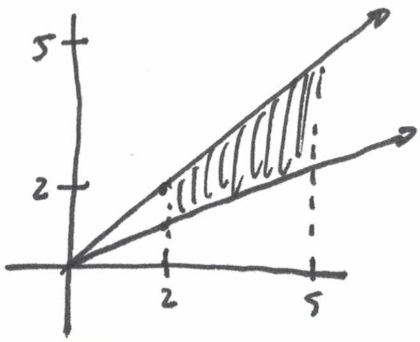
$$= -\tfrac{1}{2}\pi \left(e^0 - e^2\right)$$

$$= \tfrac{1}{2}\pi \left(e^2 - 1\right)$$

$$\boxed{\approx 10.0361}$$

---

96. Find the volume of the solid formed when the region between the graphs of the functions $y = x$ and $y = \dfrac{x}{2}$ is rotated through 2π radians about the x-axis between $x = 2$ and $x = 5$.



$$V = \pi \int_2^5 (x)^2 dx - \pi \int_2^5 \left(\tfrac{x}{2}\right)^2 dx$$

$$= \pi \int_2^5 x^2 dx - \pi \int_2^5 \tfrac{1}{4} x^2 dx$$

$$= \pi \left[\tfrac{1}{3} x^3\right]_2^5 - \pi \left[\tfrac{1}{12} x^3\right]_2^5$$

$$= \pi \left(\tfrac{125}{3} - \tfrac{8}{3}\right) - \pi \left(\tfrac{125}{12} - \tfrac{8}{12}\right)$$

$$39\pi - \tfrac{39}{4}\pi$$

$$\boxed{V = \tfrac{117}{4}\pi}$$

---

97. Find the Maclaurin series for $\ln(1 + \sin x)$ up to and including the x^2 term.

$$f(x) = \boxed{\ln(1+\sin x)}$$

$$f'(x) = \frac{1}{1+\sin x} \cdot \cos x = \boxed{\frac{\cos x}{1+\sin x}}$$

$$f''(x) = \frac{(1+\sin x)(-\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$$

$$f''(x) = \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$$

PYTHAGOREAN IDENT

$$f''(x) = \frac{-\sin x - 1}{(1+\sin x)^2}$$

$$f''(x) = \boxed{\frac{-1}{1+\sin x}}$$

$$f(0) = 0$$

$$f'(0) = \frac{1}{1} = 1$$

$$f''(0) = \frac{-1}{1} = -1$$

$$\ln(1+\sin x) = \frac{f(0)\cdot x^0}{0!} + \frac{f'(0)\cdot x^1}{1!} + \frac{f''(0)\cdot x^2}{2!}$$

$$= \cancel{\frac{0\cdot x^0}{1}} + \frac{1\cdot x}{1} + \frac{-1\cdot x^2}{2}$$

$$= \boxed{x - \tfrac{1}{2}x^2}$$

---

98. Without using the any of the shortened formulas, find the Maclaurin series for the function $f(x) = xe^x$ up to the x^4 term. Express your final answer as a series in the form $\sum a_n \cdot x^n$.

Note: You must show the correct calculations including all necessary derivatives.

$f(x) = x \cdot e^x \longrightarrow f(0) = 0$

$f'(x) = 1 \cdot e^x + x \cdot e^x = e^x(1+x) \longrightarrow f'(0) = 1$

$f''(x) = e^x(1+x) + e^x(1) = e^x(2+x) \longrightarrow f''(0) = 2$

$f'''(x) = e^x(2+x) + e^x(1) = e^x(3+x) \longrightarrow f'''(0) = 3$

$f^{(4)}(x) = e^x(3+x) + e^x(1) = e^x(4+x) \longrightarrow f^{(4)}(0) = 4$

$\vdots$

$f^{(n)}(x) = e^x(n+x)$

$f(x) = \frac{f(0)x^0}{0!} + \frac{f'(0)x^1}{1!} + \frac{f''(0)x^2}{2!} + \dots$

$\boxed{f(x) = x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4}$

---

99. Find the first three nonzero terms in the Maclaurin series for $f(x) = e^x \sin(-2x)$.

$f(x) = e^x \cdot \sin(-2x)$

$f'(x) = e^x \cdot \sin(-2x) + e^x(-2\cos(-2x))$

$f'(x) = e^x(\sin(-2x) - 2\cos(-2x))$

$f''(x) = e^x(\sin(-2x) - 2\cos(-2x))$
$\quad + e^x(-2\cos(-2x) - 4\sin(-2x))$

$f''(x) = -e^x(3\sin(-2x) + 4\cos(-2x))$

$f'''(x) = -e^x(3\sin(-2x) + 4\cos(-2x))$
$\quad - e^x(-6\sin(-2x) + 8\sin(-2x))$

$f'''(x) = -e^x(11\sin(-2x) - 2\cos(-2x))$

$f(0) = 0 \quad f'(0) = -2 \quad f''(0) = -4$
$f'''(0) = 2$

$f(x) = \frac{f(0) \cdot x^0}{0!} + \frac{f'(0) \cdot x^1}{1!} + \frac{f''(0) \cdot x^2}{2!} + \frac{f'''(0) \cdot x^3}{3!}$

$f(x) = \cancel{\frac{0 \cdot x^0}{1}} + \frac{-2 \cdot x^1}{1} + \frac{-4 \cdot x^2}{2} + \frac{2 \cdot x^3}{6}$

$\boxed{f(x) = -2x - 2x^2 + \frac{1}{3}x^3}$

---

100. a.) Find the fourth degree Maclaurin polynomial of $f(x) = \cos 2x$.

$$f(x) = \cos(2x) \qquad f(0) = 1$$

$$f'(x) = -2\sin(2x) \qquad f'(0) = 0$$

$$f''(x) = -4\cos(2x) \qquad f''(0) = -4$$

$$f'''(x) = 8\sin(2x) \qquad f'''(0) = 0$$

$$f^{(4)}(x) = 16\sin(2x) \qquad f^{(4)}(0) = 16$$

$$f(x) = \frac{f(0)\cdot x^0}{0!} + \frac{f'(0)\cdot x^1}{1!} + \frac{f''(0)\cdot x^2}{2!} + \dots$$

$$f(x) = 1\cdot\frac{x^0}{1} - 4\cdot\frac{x^2}{2!} + 16\cdot\frac{x^4}{4!}$$

ALL ODD POWER TERMS $= 0$

$$\boxed{f(x) = 1 - 2x^2 + \frac{2}{3}x^4}$$

b.) Use your answer from part (a) to approximate the value of the function at $x = 0.3$.

$$f(0.3) = 1 - 2(0.3)^2 + \frac{2}{3}(0.3)^4$$

$$f(0.3) = 1 - 0.18 + 0.0486$$

$$f(0.3) = 0.8686$$

SHORTCUT

c.) Compare the approximation from part (b) with the actual value of the function, calculating the difference between the two answers. ***Note:*** Make sure to use radians for your cos calculation.

$$\cos(0.3) = 0.9553$$

RAD

$$0.9553 - 0.8686 = \boxed{0.0867}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$$

ALT METHOD

$$\cos(2x) = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \frac{(2x)^8}{8!} - \dots$$

$$\cos(2x) = 1 - \frac{4x^2}{2} + \frac{16x^4}{24} - \frac{64x^6}{720} + \frac{512x^8}{40{,}320} - \dots$$

---

101. Find the 3rd degree Taylor series for the function $f(x) = e^{-x}$ centered about $x = \ln 2$.

$f(x) = e^{-x}$ $\quad f(\ln 2) = \frac{1}{2}$

$f'(x) = -e^{-x}$ $\quad f'(\ln 2) = -\frac{1}{2}$

$f''(x) = e^{-x}$ $\quad f''(\ln 2) = \frac{1}{2}$

$f'''(x) = -e^{-x}$ $\quad f'''(\ln 2) = -\frac{1}{2}$

$$f(x) = \frac{f(\ln 2)\cdot(x-\ln 2)^0}{0!} + \frac{f'(\ln 2)\cdot(x-\ln 2)^1}{1!} + \frac{f''(\ln 2)\cdot(x-\ln 2)^2}{2!} + \dots$$

$$\boxed{f(x) = \frac{1}{2} - \frac{1}{2}(x-\ln 2) + \frac{1}{4}(x-\ln 2)^2 - \frac{1}{12}(x-\ln 2)^3}$$

---

102. Use the first 4 terms of a Maclaurin series to approximate the integral, $\int_0^1 \sin\left(x^2\right)dx$. You may express your answer as a decimal or a fraction in reduced form.

GIVEN MACLAURIN SERIES OF $\sin x$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots$$

$$\sin(x^2) = x^2 - \frac{1}{6}x^6 + \frac{1}{120}x^{10} - \frac{1}{5040}x^{14}$$

$$\int_0^1 \sin(x^2) = \int_0^1 x^2 - \frac{1}{6}x^6 + \frac{1}{120}x^{10} - \frac{1}{5040}x^{14}\,dx$$

$$= \left[\frac{1}{3}x^3 - \frac{1}{42}x^7 + \frac{1}{1320}x^{11} - \frac{1}{75600}x^{15}\right]_0^1$$

$$= \left(\frac{1}{3} - \frac{1}{42} + \frac{1}{1320} - \frac{1}{75600}\right) - (0)$$

$$= \frac{258019}{831600} \text{ or } 0.310268157768\dots$$

$\boxed{0.310}$