IB Math HL
Intro to Complex Numbers

Name: KEY

Period: ____

(A) Polynomial Operations

- Combining like terms
- Distributive property
- FOIL method of multiplying binomials
- Multiplying polynomials with 3 or more terms

1.) $4(2x^2-7x+3)-(3x+7)(x-3)$

$8x^2-28x+12-(3x^2-9x+7x-21)$

$8x^2-28x+12-3x^2+9x-7x+21$

$\boxed{5x^2-26x+33}$

2.) $(4x^2-x+5)(3x^2+2x+10)$

	$3x^2$	$+2x$	$+10$
$4x^2$	$12x^4$	$8x^3$	$40x^2$
$-x$	$-3x^3$	$-2x^2$	$-10x$
$+5$	$15x^2$	$10x$	50

$\boxed{12x^4+5x^3+53x^2+50}$

(B) Simplifying Radicals

- Perfect square factor inside radical
- Square root in the denominator
- Fraction inside the radical
- Adding and subtracting radicals

3.) $\sqrt{480}$

$48\cdot 10$

$16\cdot 3\cdot 5\cdot 2$

$\sqrt{4\cdot 4\cdot 3\cdot 5\cdot 2}$

$\boxed{4\sqrt{30}}$

4.) $\sqrt{72}-\sqrt{32}+\sqrt{27}$

$\sqrt{36}\sqrt{2}\quad \sqrt{16}\sqrt{2}\quad \sqrt{9}\sqrt{3}$

$6\sqrt{2}-4\sqrt{2}+3\sqrt{3}$

$\boxed{2\sqrt{2}+3\sqrt{3}}$

5.) $\dfrac{21}{\sqrt{98}}$ $= \dfrac{21}{7\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}$

$\sqrt{49}\sqrt{2}$

$=\dfrac{21\sqrt{2}}{14}$

$=\boxed{\dfrac{3\sqrt{2}}{2}}$

6.) $\sqrt{\dfrac{81}{24}}$ $=\dfrac{\sqrt{81}}{\sqrt{24}}$

$=\dfrac{9}{\sqrt{24}}$

$\sqrt{4}\sqrt{6}$

$=\dfrac{9}{2\sqrt{6}}\cdot\dfrac{\sqrt{6}}{\sqrt{6}}$

$=\dfrac{9\sqrt{6}}{12}$

$=\boxed{\dfrac{3\sqrt{6}}{4}}$

(C) Radicals with Conjugates

- Simplifying with radical in denominator along with another term

7.) $\dfrac{1}{2+\sqrt{6}}\cdot\dfrac{(2-\sqrt{6})}{(2-\sqrt{6})}$

$\dfrac{2-\sqrt{6}}{4-2\sqrt{6}+2\sqrt{6}-6}$

$\boxed{\dfrac{2-\sqrt{6}}{-2}}$

or

$\dfrac{\sqrt{6}-2}{2}$

8.) $\dfrac{\sqrt{5}}{5-\sqrt{10}}\cdot\dfrac{5+\sqrt{10}}{5+\sqrt{10}}$

$\dfrac{5\sqrt{5}+\sqrt{50}}{25-10}$

$\dfrac{5\sqrt{5}+5\sqrt{2}}{15}$ $\dfrac{\div 5}{\div 5}$

$\boxed{\dfrac{\sqrt{5}+\sqrt{2}}{3}}$

9.) $\dfrac{8-\sqrt{6}}{1+\sqrt{3}}\cdot\dfrac{(1-\sqrt{3})}{(1-\sqrt{3})}$

$\dfrac{8-8\sqrt{3}-\sqrt{6}+\sqrt{18}}{1-3}$

$\dfrac{8-8\sqrt{3}-\sqrt{6}+3\sqrt{2}}{-2}$

$\boxed{\dfrac{-\sqrt{6}-8\sqrt{3}+3\sqrt{2}+8}{-2}}$

or

$\dfrac{\sqrt{6}+8\sqrt{3}-3\sqrt{2}-8}{2}$

(D) Simplifying solutions using the Quadratic Formula

- Two real solutions
- One real repeated solution
- Two imaginary solutions

Solve the following equations using the Quadratic Formula. Give the exact solutions in simplified form.

10.) $x^2 - 2x - 15 = 0$

$x = \frac{2 \pm \sqrt{4-4(1)(-15)}}{2}$

$x = \frac{2 \pm \sqrt{64}}{2}$

$x = \frac{2 \pm 8}{2}$

$\boxed{x = 5, -3}$

11.) $3x^2 + 27 = 18x$

$\frac{3x^2 - 18x + 27}{3} = \frac{0}{3}$

$x^2 - 6x + 9 = 0$

$x = \frac{6 \pm \sqrt{36 - 4(1)(9)}}{2}$

$x = \frac{6 \pm 0}{2}$

$\boxed{x = 3}$

12.) $x^2 + 2x = +12$

$x^2 + 2x + 12 = 0$

$x = \frac{-2 \pm \sqrt{4-4(1)(-12)}}{2}$

$x = \frac{-2 \pm \sqrt{+52}}{2} \rightarrow \sqrt{4}\sqrt{13}$

$x = \frac{-2 \pm 2\sqrt{13}}{2}$

$\boxed{x = -1 \pm \sqrt{13}}$

(E) Imaginary Numbers – What are they (Negative inside a root)

- The letter i as a number
- Powers of i, i^2 = -1 most important

Solve the following quadratic equations. Express your solution(s) in simplified form.

13.) $x^2 = -16$

$x = \pm\sqrt{16}$

$x = \pm\sqrt{16}\sqrt{-1}$

$\boxed{x = \pm 4i}$

14.) $4x^2 + 32 = 0$

$4x^2 = -32$

$x^2 = -8$

$x = \pm\sqrt{-8}$

$\sqrt{8}\sqrt{-1}$

$\sqrt{4}\sqrt{2}$

$x = \pm 2\sqrt{2}i$

$\boxed{x = \pm 2i\sqrt{2}}$

15.) $(x^2 - 1)(x^2 + 9) = 0$

$x^2 - 1 = 0 \quad x^2 + 9 = 0$

$x^2 = 1 \quad x^2 = -9$

$\boxed{x = \pm 1, \pm 3i}$

Express the powers of ***i*** in simplest form (no powers of ***i*** greater than 1). $i^0 = 1 \quad i^1 = i \quad i^2 = -1 \quad i^3 = -i$

16.) i^7 $7 \div 4 = 1 \text{ r. } 3$

$i^7 = i^3$

$i^3 = \boxed{-i}$

17.) $i^{10} + i^{12}$

$10 \div 4 = 2 \text{ r. } 2 \quad 12 \div 4 = 3 \text{ r. } 0$

$i^2 + i^0$

$\boxed{-1 + 1} = \boxed{0}$

18.) $i \cdot i^3 \cdot i^5 \cdot i^7$

$i^{1+3+5+7}$

$i^{16} \quad 16 \div 4 = 4 \text{ r. } 0$

$i^4 = i^0$

$\boxed{1}$

19.) $(i^{11})^5$

i^{55}

$55 \div 4 = 13 \text{ r. } 3$

i^3..

$\boxed{-i}$

(F) Complex Numbers

- Algebraic Perspective
  - Adding, subtracting and multiplying complex numbers
  - Dividing complex numbers using complex conjugates

Determine the real and imaginary part of the following expressions.

20.) $1-4i+7+8i-11$

$\boxed{-3+4i}$

21.) $2(3+2i)-7(5+6i)$

$6+4i-35-42i$

$\boxed{-29-38i}$

22.) $(4-9i)(7-8i)$

$28-32i-63i+72i^2$

$28-72-32i-63i$

$\boxed{-44-95i}$

23.) $\dfrac{2}{1+i}\dfrac{(1-i)}{(1-i)}$

$\dfrac{2(1-i)}{1-i^2}$

$\dfrac{2-2i}{1+1}$

$\dfrac{2-2i}{2}$

$\boxed{1-i}$

24.) $\dfrac{6i}{4-3i}\dfrac{(4+3i)}{(4+3i)}$

$\dfrac{24i+18i^2}{16-9i^2}$

$\dfrac{24i-18}{16+9}$

$\boxed{\dfrac{-18+24i}{25}}$

25.) $\dfrac{6-5i}{3-2i}\dfrac{(3+2i)}{(3+2i)}$

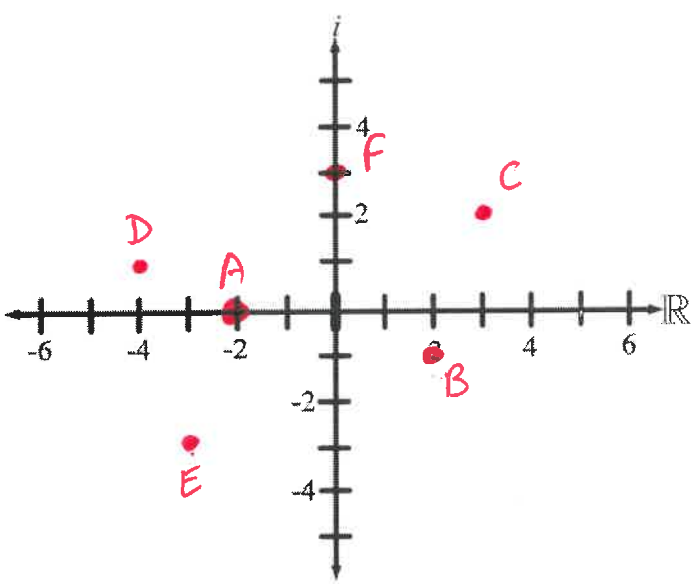
$\dfrac{18+12i-15i-10i^2}{9-4i^2}$

$\dfrac{18+10+12i-15i}{9+4}$

$\boxed{\dfrac{28-3i}{13}}$

- Graphical Perspective
  - Argand diagram with real and imaginary axes
  - *Complex numbers as vectors (skipping for now)*
  - *Tip to tail method of adding vectors (skipping for now)*

26.) Plot the following complex numbers on the complex plane (Argand diagram) below.



-2 A

$2-i$ B

$3+2i$ C

$-4+i$ D

$-3-3i$ E

$3i$ F