IB Math HL2
Integration Part 1 – Review

Name: KEY

Derivatives and Integration

1.) Given $f(x)=(4x+7)^5$ find $f'(x)$.

$f'(x) = 5(4x+7)^4 \cdot 4$

$f'(x) = 20(4x+7)^4$

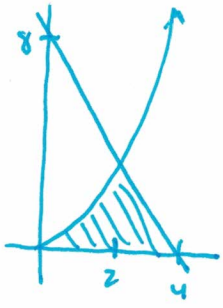
2.) $\int (4x+7)^5 \cdot dx$

$(4x+7)^6 \cdot \frac{1}{6} \cdot \frac{1}{4}$

$\frac{1}{24}(4x+7)^6 + C$

Areas

3.) Find the area enclosed by the graphs of $y=x^2$, $y=8-2x$, and the x-axis in quadrant 1.



$x^2 = 8-2x$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x=-4, 2$

$x=2$

$A = \int_0^2 x^2\,dx + \int_2^4 8-2x\,dx$

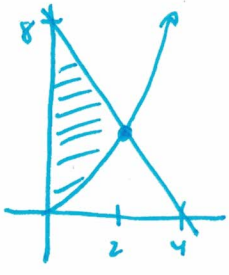
$= \left[\frac{1}{3}x^3\right]_0^2 + \left[8x-x^2\right]_2^4$

$= \left(\frac{8}{3}-0\right) + (32-16-(16-4))$

$= \frac{8}{3} + 4$

$A = 6\frac{2}{3}$ or $\frac{20}{3}$

4.) Find the area enclosed by the graphs of $y=x^2$, $y=8-2x$, and the y-axis in quadrant 1.



SEE FACTORING FROM #3.

$A = \int_0^2 -x^2-2x+8\,dx$

$A = \left[-\frac{1}{3}x^3 - x^2 + 8x\right]_0^2$

$A = \left(-\frac{8}{3}+4+16\right) - 0$

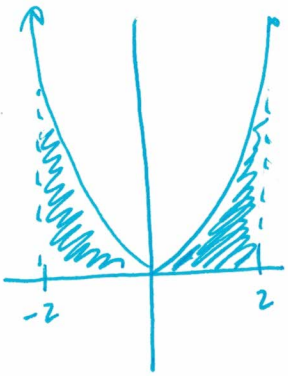
$A = \frac{28}{3}$ or $9.\overline{3}$

$A \neq 36$

NOT $\int_{-4}^2 -x^2-2x+8\,dx$

THIS INCLUDES QUADRANT 2

5.) The area between the curve $y = ax^2$ and the x-axis between -2 and 2 is 20. Find the value of a.



$$20 = 2\int_0^2 ax^2\,dx$$

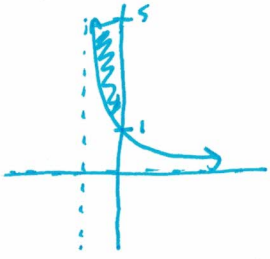
$$20 = 2\left[\tfrac{1}{3}ax^3\right]_0^2$$

$$20 = 2\left(\tfrac{8}{3}a\right)$$

$$20 = \tfrac{16}{3}a$$

$a = \frac{15}{4}$ or 3.75

6.) Find the area enclosed by the graph of $y = \dfrac{1}{x+1}$, the y-axis, and the line $y = 5$.



$$y = \frac{1}{x+1}$$

$$x+1 = \frac{1}{y}$$

$$x = \frac{1}{y} - 1$$

$$A = \int_1^5 \frac{1}{y} - 1\,dy$$

$$A = \left[\ln|y| - y\right]_1^5$$

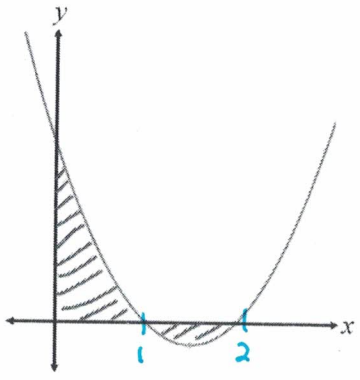
$$A = (\ln 5 - 5) - (\ln 1 - 1)$$

$$A = \ln 5 - 4$$

$$A = -2.391$$

$A = 2.391$

7.) The diagram below shows the graph of $y = x^2 - 3x + 2$. Find the area of the shaded region.



$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$x = 1, 2$$

$$A = \int_0^1 x^2 - 3x + 2\,dx + \int_1^2 x^2 - 3x + 2\,dx$$

ABOVE X-AXIS; BELOW X-AXIS → ABS

$$A = \left[\tfrac{1}{3}x^3 - \tfrac{3}{2}x^2 + 2x\right]_0^1 + \left|\left[\tfrac{1}{3}x^3 - \tfrac{3}{2}x^2 + 2x\right]_1^2\right|$$

$$A = \left[\left(\tfrac{1}{3} - \tfrac{3}{2} + 2\right) - (0)\right] + \left|\left[\left(\tfrac{8}{3} - 6 + 4\right) - \left(\tfrac{1}{3} - \tfrac{3}{2} + 2\right)\right]\right|$$

$$A = \left(\tfrac{5}{6}\right) + \left|\left(\tfrac{2}{3} - \tfrac{5}{6}\right)\right|$$

$$A = \tfrac{5}{6} + \tfrac{1}{6}$$

$A = 1$