Worksheet #1: Review of Differentiation and Basic Integration Skills

The following worksheet is designed to help review and/or sharpen your ability to differentiate and integrate functions encountered in a typical Calculus 1 course. Section IV also addresses some good conceptual questions about the relationship between a fiction and its antiderivatives.

I. Differentiation Practice

Differentiate the following functions.

a) $y = (2x-7)^4$ *b)* $y = e^{\frac{x}{4}}$ *c)* $y = 7x^4 - 3\sqrt[5]{x} + \dfrac{2}{5x^2}$

d) $y = \ln(2x + \cos x)$ *e)* $y = 2xe^{-x}$ *f)* $y = \dfrac{\tan(3x)}{\sqrt{4-x}}$

g) $y = \csc\left(e^{4x}\right)$ *h)* $y = [\ln(4x^3 - 2x)]^3$ *i)* $y = e^{4\sqrt{x}}$

j) $y = 4e^{x\sin x}$ *k)* $y = 6x^9 - \dfrac{1}{8x^4} + \dfrac{2}{\sqrt[3]{2x-1}}$ *l)* $y = \dfrac{2}{(3x^2-1)^2}$

II. Integration Practice

Compute the following integrals. If an integral cannot be algebraically reduced to one of the basic functions (powers of x, trig functions, exponentials, etc) that can be easily integrated, state so!

a) $\displaystyle\int \left(3x^4 - \sqrt[3]{x^2} + \frac{2}{\sqrt[7]{x}}\right) dx$ *b)* $\displaystyle\int e^{x^2}\, dx$ *c)* $\displaystyle\int_0^{\pi/6} 4\sin(2x)\, dx$

d) $\displaystyle\int e^{-\frac{x}{3}}\, dx$ *e)* $\displaystyle\int \ln x\, dx$ *f)* $\displaystyle\int \frac{4x^3 - 3x}{2x^2}\, dx$

g) $\displaystyle\int \left(\sec(4x)\tan(4x) + 3\sec^2\frac{x}{5}\right) dx$ *h)* $\displaystyle\int_1^4 (\sqrt{x}-1)^2\, dx$ *i)* $\displaystyle\int_0^1 \sqrt{e^{3x}}\, dx$

j) $\displaystyle\int \cot^2(3x)\sec^2(3x)\, dx$ *k)* $\displaystyle\int \cos\sqrt{x}\, dx$ *l)* $\displaystyle\int \frac{2}{(3x)^2}\, dx$

III. Miscellaneous

The following questions help dispel common integration errors and allow for one to gain some insight as to why these incorrect methods fail.

1. Consider the function $f(x) = e^{2x}$. We know that $\dfrac{d}{dx}\left(e^{2x}\right) = 2e^{2x}$ by the Chain Rule, and this lets us easily conclude that $\int e^{2x}\, dx = \frac{1}{2}e^{2x} + C$. This could of course be verified by u-substitution (if you know/remember this technique), but can also be understood the following way:

Function Type	Integration Rules (c is the constant of variation)	Notes
1.) Constant	$\int m\cdot dx = mx + c$ where m is a constant	
2.) Constant Multiple	$\int k\cdot f(x) = k\cdot\int f(x)$ where k is a constant	Only works if k is constant.
3.) Power	$\int x^n\cdot dx = \dfrac{x^{n+1}}{n+1} + c, n \neq -1$	Must ignore $n \neq -1$ to avoid division by zero.
4.) Power of -1	$\int x^{-1}\cdot dx = \int \frac{1}{x}\cdot dx = \ln\lvert x\rvert + c$	This rule takes care of the $n \neq -1$ exception for Power.
5.) Sum and Difference	$\int [f(x) \pm g(x)]\cdot dx = \int f(x)\cdot dx \pm \int g(x)\cdot dx$	This rule also applies if there are more than 2 terms.
6.) e^x	$\int e^x\cdot dx = e^x + c$	Specific instance of Exponential where $\ln e = 1$.
7.) Exponential	$\int m^x\cdot dx = \dfrac{m^x}{\ln m} + c$, m is a constant	General rule for exponential.
8.) Trig	$\int \sin x\cdot dx = -\cos x + c$	
	$\int \cos x\cdot dx = \sin x + c$	
	$\int \tan x\cdot dx = -\ln\lvert\cos x\rvert + c = \ln\lvert\sec x\rvert + c$	We can derive this later in the course.
9.) Inverse Trig	$\int \dfrac{1}{\sqrt{a - x^2}} dx = \arcsin\left(\dfrac{x}{a}\right) + c$	Most instances will involve $a = 1$ and will be simpler to integrate.
	$\int \dfrac{-1}{\sqrt{a - x^2}} dx = \arccos\left(\dfrac{x}{a}\right) + c$	Most instances will involve $a = 1$ and will be simpler to integrate.
	$\int \dfrac{1}{a^2 + x^2} dx = \dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right) + c$	Most instances will involve $a = 1$ and will be simpler to integrate.

Differentiation Rules	Function	Derivative Rule
1.) Constant	$f(x) = a$, a is a constant	$f'(x) = 0$
2.) Constant Multiple	$f(x) = a\cdot g(x)$, a is a constant	$f'(x) = a\cdot g'(x)$
3.) Power	$f(x) = x^a$, $a \in \mathbb{R}$	$f'(x) = a\cdot x^{a-1}$
4.) Sum and Difference	$f(x) = g(x) \pm h(x)$	$f'(x) = g'(x) \pm h'(x)$
5.) Chain Rule	$f(x) = g(h(x))$	$f'(x) = g'(h(x))\cdot h'(x)$
6.) Product Rule	$f(x) = g(x)\cdot h(x)$	$f(x) = g'(x)\cdot h(x) + g(x)\cdot h'(x)$
7.) Quotient Rule	$f(x) = \dfrac{g(x)}{h(x)}$	$f'(x) = \dfrac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$
8.) Exponential	$f(x) = a^x$, a is a constant	$f'(x) = \ln a\cdot a^x$
9.) e^x	$f(x) = e^x$	$f'(x) = \ln e\cdot e^x = e^x$
10.) Logarithmic	$f(x) = \log_b x$	$f'(x) = \dfrac{1}{x\ln b}$
11.) Natural Log	$f(x) = \ln x = \log_e x$	$f'(x) = \dfrac{1}{x\ln e} = \dfrac{1}{x}$
12.) Trig	$f(x) = \sin x$	$f'(x) = \cos x$
	$f(x) = \cos x$	$f(x) = -\sin x$
	$f(x) = \tan x$	$f'(x) = \sec^2 x$
	$f(x) = \csc x$	$f'(x) = -\csc x\cot x$
	$f(x) = \sec x$	$f'(x) = \sec x\tan x$
	$f(x) = \cot x$	$f(x) = -\csc^2 x$
13.) Inverse Trig	$f(x) = \arcsin x$ $g(x) = \arcsin\left(\dfrac{x}{a}\right)$	$f'(x) = \dfrac{1}{\sqrt{1-x^2}}$ $g'(x) = \dfrac{1}{\sqrt{a^2 - x^2}}$
	$f(x) = \arccos x$ $g(x) = \arccos\left(\dfrac{x}{a}\right)$	$f'(x) = -\dfrac{1}{\sqrt{1-x^2}}$ $g'(x) = -\dfrac{1}{\sqrt{a^2 - x^2}}$
	$f(x) = \arctan x$ $g(x) = \arctan\left(\dfrac{x}{a}\right)$	$f'(x) = \dfrac{1}{1+x^2}$ $g'(x) = \dfrac{a}{a^2 + x^2}$