4T - Implicit Differentiation

Warmup

Find the inverse of the function below.

$$y=\frac{x+3}{4-x}$$

$$x=\frac{y+3}{4-y}$$

$$x(4-y)=y+3$$

$$4x-xy=y+3$$

$$-xy-y=-4x+3$$

$$y(-x-1)=-4x+3$$

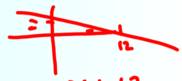
$$y=\frac{-4x+3}{-x-1}$$

$$y=-\frac{-4x+3}{x+1}$$

$$y=\frac{4x-3}{x+1}$$

Explicit Functions

Explicit Function = Dependent variable (y) is defined in terms of the independent variable (x).



$$y=-\frac{1}{6}x+2 \rightarrow 6y=-x+12$$

$$\boxed{x+6y=12}$$

Nearly all of the functions we have differentiated thus far have been explicit functions. These are much easier to differentiate. But, not all functions or equations will be written this way.

$$x+6y=12$$

Example 1:

Find the gradient of the circle $x^2+y^2=9$ at x=3.

$$x^2+y^2=9$$

$$2x\cdot\cancel{\frac{dx}{dx}}+2y\cdot\frac{dy}{dx}=0$$

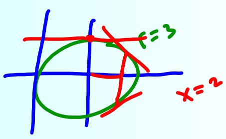
$$2x+2y\cdot\frac{dy}{dx}=0$$

$$y=x^2+2x$$

$$\frac{dy}{dx}=\sim$$

$$\frac{dy}{dx}=-\frac{2x}{2y}$$

$$\frac{dy}{dx}=\frac{-x}{y}\;\frac{0}{3}\;\frac{-3}{0}$$



Explicit Functions

In example 1, we had two issues arise.

1. The equation was not written explicitly since it was not solved for y in terms of x.

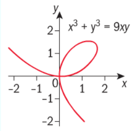
2. The equation was not a function. Although this is an issue, we were easily able to adjust the equation by solving for y, thus creating two separate functions that modeled our original circle equation.

$$y=\sqrt{9-x^2} \qquad y=-\sqrt{9-x^2}$$

4T - Implicit Differentiation

Differentiate the Folium of Descartes,

$$x^3 + y^3 = 9xy.$$



The Folium of Descartes

This equation is not a function, which we will address later. But more importantly, how do we solve for y in terms of x?

Simple answer: We can't!

Implicit Differentiation

- Allows you to differentiate functions that are not written explicitly.
- Also allows you to differentiate equations that are not functions.

Steps

- All previous differentiation rules still apply
- You are now differentiating ALL variables in terms of x
- When you differentiate anything with y in it, you must add a new term (will discuss on next slide).

Example 2:
Use implicit differentiation on the equation below.

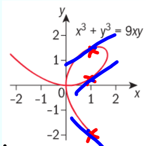
$$y^2 = 4x$$

$$2y \cdot \frac{dy}{dx} = 4$$

$$\frac{dy}{dx} = \frac{2}{y}$$

Example 3:
Differentiate the Folium of Descartes,

$$x^3 + y^3 = 9xy.$$



The Folium of Descartes

$$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 9 \cdot y + 9x \cdot 1 \cdot \frac{dy}{dx}$$

$$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 9y + 9x \cdot \frac{dy}{dx}$$

$$3y^2 \cdot \frac{dy}{dx} - 9x \cdot \frac{dy}{dx} = 9y - 3x^2$$

$$\frac{dy}{dx}(3y^2 - 9x) = 9y - 3x^2$$

$$\frac{dy}{dx} = \frac{9y - 3x^2}{3y^2 - 9x}$$

$$\frac{dy}{dx} = \frac{3y - x^2}{y^2 - 3x}$$

$$x^3 + y^3 = 9xy$$

$$1 + y^3 = 9y$$

$$y^3 - 9y + 1 = 0$$

4T - Implicit Differentiation

Example 4:
Find the slope of the tangent line to the graph below at *(-4,1)*.

$$x^2y^3 + xy = 12$$

$$x^2y^3 + xy = 12$$

$$2x \cdot y^3 + x^2 \cdot 3y^2 \cdot \frac{dy}{dx} + y + x \cdot \frac{dy}{dx} = 0$$

$$2xy^3 + 3x^2y^2 \cdot \frac{dy}{dx} + y + x \cdot \frac{dy}{dx} = 0$$

$$3x^2y^2 \cdot \frac{dy}{dx} + x \cdot \frac{dy}{dx} = -y - 2xy^3$$

$$\frac{dy}{dx}\left(3x^2y^2 + x\right) = -y - 2xy^3$$

$$\frac{dy}{dx} = \frac{-y - 2xy^3}{3x^2y^2 + x}$$

Example 5:
The point $P(2,m)$, where $m<0$, lies on the curve $2x^2y + 3y^2 = 16$.

a.) Calculate the value of m.

b.) Find the gradient of the normal to the tangent at P.