**Topic List**

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| **Unit 01 – Limits, Continuity, and Differentiability*** Algebraic Limits
* Squeeze Theorem
* Limits with piece-wise functions
* L’Hoptial’s Rule
* Continuity
* Differentiability
* IVT (Intermediate Value Theorem)
* Rolle’s Theorem
* MVT (Mean Value Theorem)

**Unit 02 – Improper Integrals*** Fundamental Theorem of Calculus
* Improper integral as limit
* Comparison Tests
* Upper and Lower Reimann Sums

**Unit 03 – Infinite Series and Convergence** * Infinite Series in Summation Notation
* Partial Sums
* Convergence Tests
* Power Series
* Radius and Interval of Convergence

**Unit 04 – Taylor and Maclaurin Series*** Maclaurin Series
* Truncated Maclaurin Polynomials
* Lagrange Form of Error Term
* Maclaurin Series of Composite Functions
* Taylor Series

**Unit 05 – Differential Equations*** Simple differential equations
* Separation of variables
* Homogenous differential equations
* Linear differential equations
* General vs. particular solution
* Implicit vs. explicit form
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**Unit 01 – Limits, Continuity, and Differentiability**

* Algebraic Limits
	+ Simplifying using factoring, conjugates, and other algebraic techniques
	+ Long form of derivative: 
* Squeeze Theorem
	+  , If and  approach the same limit, then approaches same limit
* Limits with piece-wise functions
	+ Have to approach the same y-value at the interval changes
	+ Limit exists only if LHL = RHL (Left-hand limit = right-hand limit,  )
* L’Hoptial’s Rule
	+  must be in the form  or 
	+ 
	+ May be repeated more than once, but must be  or  at each step
* Continuity at a point
	+ Function at x=c () must exist
	+ Limit must exist ()
	+ Function value and limit from above steps must be same value
* Differentiability at a point
	+ Must be continuous
	+ Must not have any vertical tangents (  at x=c must be defined)
	+ No “sharp turns” – Check derivative coming from each side at x=c to make sure they are equal
* IVT (Intermediate Value Theorem)
	+ There exists 2 values such that  and  (Can be flipped as long as one >, one <)
	+ Must exist a point  such that 
	+ Usually used for checking existence of a root/x-intercept
* Rolle’s Theorem
	+ Function must be continuous on  and differentiable on 
	+ If  , then there must exist  such that 
	+ Used to show that a max/min does or does not exist on an interval
	+ Can prove existence or lack or multiple root of a polynomial
* MVT (Mean Value Theorem)
	+ Function must be continuous on  and differentiable on 
	+ There must exist a point  such that 
	+ Must be some point on interval where slope of tangent is same as slope between endpoints

**Unit 02 – Improper Integrals**

* Fundamental Theorem of Calculus
	+ Essentially allows for definite integration
	+  where 
* Improper integral as limit
	+ 
	+ Use integration techniques from Calculus 1
* Comparison Tests for Improper Integrals
	+ If  for all  , then
		- If is convergent, then is convergent
		- If is divergent, then is divergent
* Upper and Lower Reimann Sums
	+ Gives upper and lower boundaries on the value of infinite series that may or may not be able to be evaluated
	+ 2 similar forms depending on whether the function is decreasing or increasing
	+ Decreasing Function for all , then
		- 
	+ Increasing Function for all , then
		- 
	+ Might be able to evaluate the outer sums using infinite geometric series formulas
		-  , where  is the common ratio

**Unit 03 – Infinite Series and Convergence**

* Infinite Series in Summation Notation
	+ Upper boundary becomes infinity
	+ 
* Partial Sums
	+ Different name for sum of the first n terms in an infinite series
	+ N-th partial sum is defined as 
	+ Same as  for arithmetic series
	+ Same as  for geometric series
* Convergence Tests



* Power Series
	+ Infinite Series in the form 
	+ When  , reduces to 
	+ Same form we use for Maclaurin and Taylor Series expansions
* Radius and Interval of Convergence
	+ ***Use ratio test***
	+ If  then the series converges for all 
	+ If  then the series converges for only at one specific value
	+ Radius of convergence – Simple number, interval of convergence length divided by 2
	+ Interval of convergence
		- Must test outer boundaries for convergence
		- Usually, one becomes the less strict alternating series test

**Unit 04 – Taylor and Maclaurin Series**

* Maclaurin Series
	+ Centered around  , so every part is evaluated at 0.
	+ Power series in form 
	+ Can only be used if the function exists at for all derivatives of 
* Truncated Maclaurin Polynomials
	+ Same as above Maclaurin expansion, but only taking the first n terms for an n-th degree Maclaurin polynomial
* Lagrange Form of Error Term
	+ 
	+  is defined as the error term, or how close you are to the actual sum by only taking n terms
	+ , just assume for this test that 
* Maclaurin Series of Composite Functions
	+ Use the common Maclaurin shortcuts to perform expansions without doing the long, derivative method
* Taylor Series
	+ Centered around  to gain a better estimate
	+ Power series in form 
	+ Given in formula booklet: 

**Unit 05 – Differential Equations**

* Simple differential equations
	+  , solve using simple integration techniques
* Separation of variables
	+ Separate so  and  on left and  and  on right
	+ Integrate per usual
* General vs. particular solution
	+ General solution leaves the +C as you would for an indefinite integral
	+ Particular solution will give values for x and y and solve for C
* Implicit vs Explicit Form
	+ Implicit form allows you to leave as is without much simplification
	+ Explicit requires you to solve for y
	+ Leave in implicit form unless otherwise asked
* Note: You may mix general/particular with implicit/explicit in any way (So implicit with general is one)
* Homogenous differential equations
	+ Can be written in the form
	+ Once rewritten, you perform substitutions which will allow you to solve using separation of variables
	+ Main substitution: 
	+ Derivative of the above equation: 
	+ General/particular and implicit/explicit still apply
* Linear differential equations
	+ Written in the form: 
	+ Use the integrating factor: 
	+ Multiply through the top form by the integrating factor, then do reverse product rule