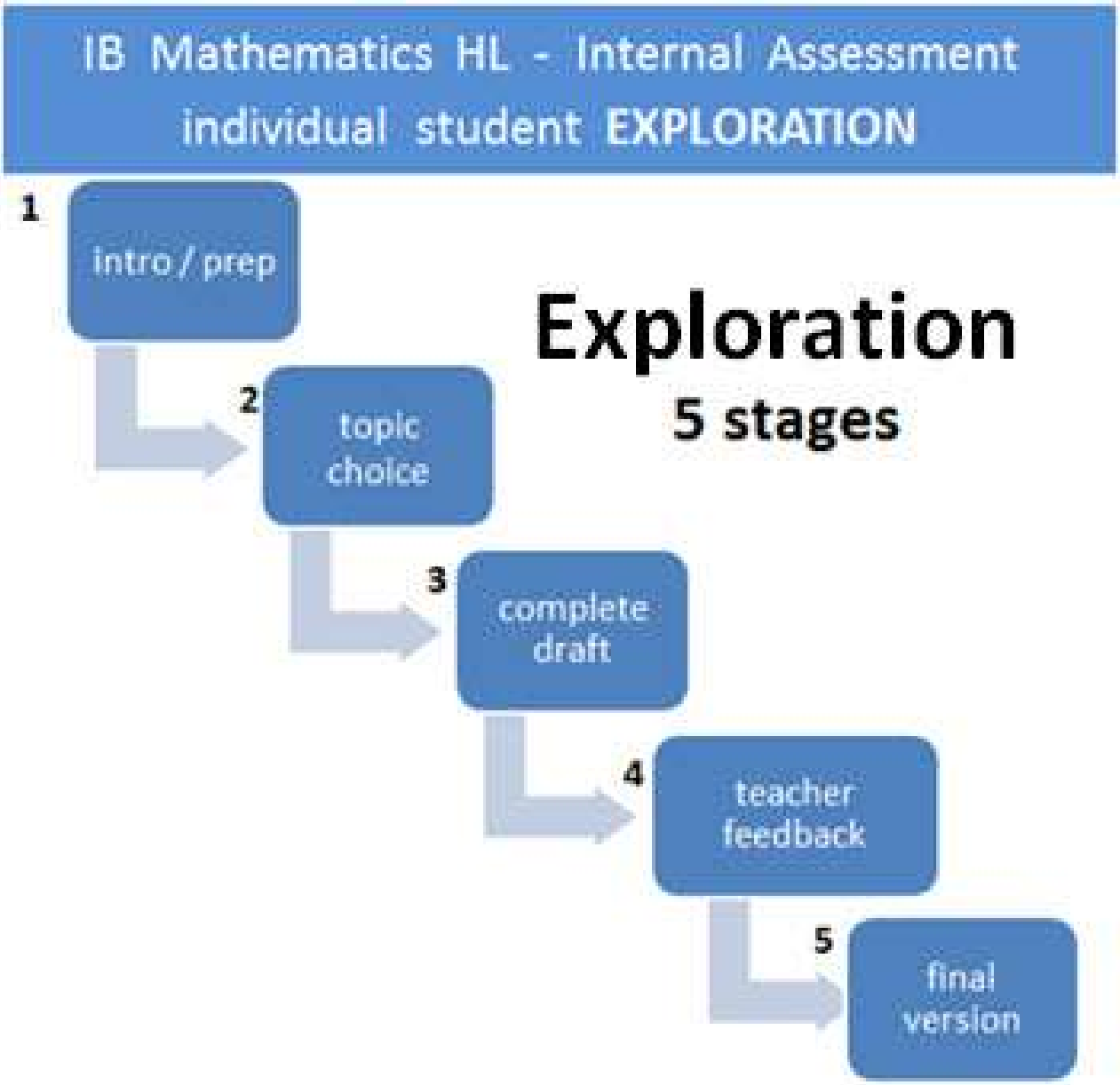
IB Mathematics Higher Level

Internal Assessment - The Exploration

~ Student Guide ~



Timeline:

Stage	Description	Start/Due Date
1. Introduction/Topic Brainstorm	Student research and brainstorm of interested topics.	May-Aug 2020
2. 3 Topic Choices	Brainstorm and return with 3 potential topics and a short description for each.	Last Week of August 2020
3. Final Topic with Rough Outline	Using your topic and abstract, generate an outline with an essential question of exploration, including at least 3 subtopics and info gathered from research. You must include a source page, but MLA format is not required.	Mid Septemeber 2020
4. Rough Draft #1	Progress check where you should be roughly halfway through your paper	Halloween-Thanksgiving 2020
5. Final Draft #2	Full length rough draft submitted prior to winter break for review. This should be as close to a final draft as possible, with title page, works citied, and table of contents if necessary.	Late December 2020 (Last week before winter break)
6. Teacher Feedback	Returned rough drafts with teacher comments and suggestions.	Mid Jan 2021
7. Final Draft (Ready for IB Submission)	Final IB submission with properly formatted title page and works cited.	Early March 2021

The Exploration – Top Tips

1. Choose a topic in consultation with your teacher that: (i) you're interested in, (ii) involves math at a level suitable for Math HL, (iii) is narrow enough for 8-14 pages, (iv) has opportunities for personal engagement.

2. Your Exploration must have an aim or objective which involves doing some mathematics. It is important to maintain a focus on the overall aim/objective and a focus on mathematical concepts and methods.

3. Although all the work on your Exploration must be your own, do not hesitate to ask your teacher for advice and feedback at any stage. Your teacher will provide written and/or oral feedback on your draft.

4. Be sure you fully understand the expectations of the five assessment criteria, and refer back to them while you are planning and writing your Exploration.

5. The Exploration is an opportunity to complete a significant assessment item (20% of IB score) while **not** under the pressure of timed exam conditions. Take advantage of the opportunity by following instructions, meeting deadlines, engaging & reflecting in your own way, and enjoying some math you are interested in.

Introduction into the IB Mathematics HL Internal Assessment (IA)

What is Internal Assessment in IB Mathematics Higher Level?

Internal Assessment (IA) in Maths HL consists of a single, internally assessed component (i.e. marked by the teacher) called a mathematical exploration (or just the "Exploration"). The Exploration contributes **20%** to your overall IB score for the course.

What is the Exploration?

Your Exploration is a written report (8-14 pages without title page, works cited, and any appendices) involving a mathematical topic that interests you. You will choose a topic in consultation with your teacher after conducting your own research.

How is the Exploration assessed?

Your Exploration will earn a score out of 20 marks based on the following five criteria. Further details for each criterion and guidance for addressing them is provided later in this guide. A more detailed description of each of these categories is included at the end of this document.

Criterion A	Max **4** Marks	**Presentation**
Criterion B	Max **4** Marks	**Mathematical Communication**
Criterion C	Max **3** Marks	**Personal Engagement**
Criterion D	Max **3** Marks	**Reflection**
Criterion E	Max **6** Marks	**Use of Mathematics**

Some important points to consider:

♦ In your Exploration you need to write ***about mathematics*** and not just ***do mathematics***.

♦ Any idea, method, content, etc that is not your own must be **cited** at the point in the Exploration where it is used. Just listing your sources in a bibliography is not enough and may lead to the IB deciding that malpractice has occurred.

♦ The Exploration is an opportunity for you to learn more about a mathematical topic in which you are genuinely interested. You will be rewarded (**personal engagement**) for explaining your interest in the topic, and for demonstrating curiosity, creativity & independent thinking.

♦ You will be rewarded (**reflection**) for expressing what you think about the mathematics you are exploring. You should endeavor to pose your own questions and try to answer them using suitably sufficient level of mathematical ideas and procedures.

♦ You will be required to submit a **complete draft** of your Exploration – containing an introduction, conclusion and all planned content to sufficiently address all five criteria. You will receive feedback on the draft and then be given an opportunity to revise it to submit a final version.

♦ All of the work you do on your Exploration must be your own.

IB Mathematics HL Internal Assessment (IA) Frequently Asked Questions

What is the difference between a mathematical exploration and an extended essay in mathematics?

The criteria are completely different. It is intended that the exploration is to be a much less extensive piece of work than a mathematics extended essay. The intention is for students to "explore" an idea rather than have to do the formal research demanded in an extended essay.

How long should it be?

It is difficult to be prescriptive about mathematical writing. However, the *Mathematics SL guide* and the *Mathematics HL guide* state that 8-14 pages should be appropriate. Some papers may be a bit longer due to graphs, tables, charts, and other additional items. Even with these, a paper should not exceed 20 pages. A common failing of mathematical writing is excessive repetition, and this should be avoided, as such explorations will be penalized for lack of conciseness. However, it is recognized that some explorations will require the use of several diagrams, which may extend them beyond the page limit.

How long should it take?

It is difficult to give a single answer. However, the guideline of 10 hours class time with approximately the same amount of time outside class should suffice for students to develop their ideas and complete the exploration.

Does the exploration need a title?

It is good practice to have a title for all pieces of work. If the exploration is based on a stimulus, it is recommended that the title not just be the stimulus. Rather, the title should give a better indication of where the stimulus has taken the student. For example, rather than have the title "water", the title could be "Water—predicting storm surges".

Can students in the same school/class use the same title for the exploration?

Yes, but the explorations must be different, based on the avenues followed by each student. As noted above, the title should give an idea of what the exploration is about. Group work is not allowed.

Can students in the same school/class use the same stimulus?

Yes, this is permissible. However, the stimuli are intended to be broad themes around which a variety of foci could develop. It is therefore expected that, even if students use the same stimuli, the resulting explorations will be very different.

How many explorations should be done by a student during the course?

The exploration is a significant piece of work and, as such, the advice would be that there is no necessity to undertake more than one during the course. However, in line with the "Approaches to the teaching and learning of mathematics SL/HL" section of the two guides, students should be given many opportunities to use modeling and investigative techniques to develop the sorts of skills necessary to perform well in the exploration.

Should the scope and sequence of the HL course be influenced by the exploration?

Ideally, it should not be. It is intended that the exploration should be a natural opportunity to develop ideas that students have become familiar with as a part of the course. However, if it is felt that particular skills are likely to be needed in order for students to undertake the exploration successfully, then a teacher or school may wish to consider this when deciding on the teaching sequence.

How much help can a teacher give the student in finding a topic/focus for their exploration?

The role of the teacher here is to provide advice to the student on choosing the topic, and there is no set limit to the amount of help a teacher can give in this respect. However, if the student has little or no input into the decision about which focus to choose, then it is unlikely that he or she will be able to explore the ideas successfully in order to generate a good exploration.

How much help can the teacher give to the student with the mathematical content of the exploration?

If a student needs help with the revision of a particular topic because they are having some problems using this in their exploration, then it is permissible (indeed, this is good practice) for the teacher to give this help. However, this must be done in such a way that is not directly connected with the exploration.

Can the students use mathematics other than that they have done in class?

Yes, but this must be clearly explained and referenced, and teacher comments should clarify this.

Can students use mathematics that is outside the syllabus?

Yes, as long as the mathematics used is relevant. However, this is not necessary to obtain full marks.

What is the difference between criterion A (Presentation) and criterion B (Mathematical Communication)?

Presentation is focusing on the overall organization and coherence of the exploration, whereas Mathematical Communication focuses on the appropriateness of the mathematics. An exploration that is logically set out in terms of its overall structure could score well in criterion A despite using inappropriate mathematics. Conversely, an exploration that uses appropriate diagrams and technology to develop the ideas could score well in criterion B but poorly in criterion A because it lacked a clear aim or conclusion, for example.

What is personal engagement?

The exploration is intended to be an opportunity for students to use mathematics to develop an area of interest to them rather than merely to solve a problem set by someone else. Criterion C (personal engagement) will be looking at how well the student is able to demonstrate that he or she has “made the exploration their own” and expressed ideas in an individual way. These can be expressed through creative topic selection, unique explorations of common topics, or even extensions on topics already covered.

What is the difference between precise and correct?

As outlined in criterion E (use of mathematics), “precise” mathematics requires absolute accuracy with appropriate use of notation. “Correct” mathematics may contain the occasional error as long as it does not seriously interfere with the flow of the work or give rise to conclusions or answers that are clearly wrong.

What is a complete exploration?

In a complete exploration, all steps are clearly explained without detracting from its conciseness.

Maths SL & HL Internal Assessment (IA)
200 Exploration Ideas/Topics

Algebra & Number Theory

Modular arithmetic
Goldbach's conjecture
Probabilistic number theory
Applications of complex numbers
Diophantine equations
Continued fractions
General solution of a cubic equation
Applications of logarithms
Polar equations
Patterns in Pascal's triangle
Finding prime numbers
Random numbers
Pythagorean triples
Mersenne primes
Magic squares & cubes
Loci and complex numbers
Matrices and Cramer's rule
Divisibility tests
Egyptian fractions
Complex numbers & transformations
Euler's identity: $e^{i\pi} + 1 = 0$
Chinese remainder theorem
Fermat's last theorem
Natural logarithms of complex numbers
Twin primes problem
Hypercomplex numbers
Diophantine application: Cole numbers
Odd perfect numbers
Euclidean algorithm for GCF
Palindrome numbers
Factorable sets of integers of the form $ak + b$
Algebraic congruences
Inequalities related to Fibonacci numbers
Combinatorics – art of counting
Boolean algebra
Graphical representation of roots of complex numbers
Roots of unity

Statistics & Modelling

Traffic flow
Logistic function and constrained growth
Modelling growth of tumours
Modelling epidemics/spread of a virus
Modelling the shape of a bird's egg
Correlation coefficients
Central limit theorem
Modelling change in record performances for a sport
Hypothesis testing
Modelling radioactive decay
Least squares regression
Regression to the mean
Modelling growth of computer power

Geometry

Non-Euclidean geometries
Cavalieri's principle
Packing 2D and 3D shapes
Ptolemy's theorem
Hexaflexagons
Heron's formula
Geodesic domes
Proofs of Pythagorean theorem
Minimal surfaces & soap bubbles
Tesseract – a 4D cube
Map projections
Tiling the plane – tessellations
Penrose tiles
Morley's theorem
Cycloid curve
Symmetries of spider webs
Fractal tilings
Euler line of a triangle
Fermat point for polygons & polyhedra
Pick's theorem & lattices
Properties of a regular pentagon
Conic sections
Nine-point circle
Geometry of the catenary curve
Regular polyhedra
Euler's formula for polyhedra
Eratosthenes' - measuring earth's circumference
Stacking cannon balls
Ceva's theorem for triangles
Constructing a cone from a circle
Conic sections as loci of points
Consecutive integral triangles
Area of an ellipse
Mandelbrot set and fractal shapes
Curves of constant width
Sierpinksi triangle
Squaring the circle
Polyominoes
Reuleaux triangle
Architecture and trigonometry
Spherical geometry

Calculus/Analysis & Functions

Mean Value theorem
Torricelli's trumpet (Gabriel's horn)
Integrating to infinity
Applications of power series
Newton's law of cooling
Fundamental theorem of calculus
Brachistochrone (min.time) problem
Second order differential equations
l'Hopital's rule and evaluating limits
Hyperbolic functions
The harmonic series
Torus – solid of revolution

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Probability & Probability Distributions
The Monty Hall problem
Monte Carlo simulations
Random walks
Insurance and calculating risks
Poisson distribution and queues
Determination of π by probability
Lotteries
Bayes' theorem
Birthday paradox
Normal distribution and natural phenomena

Games & Game Theory
The prisoner's dilemma
Sudoku
Gambler's fallacy
Poker and other card games
Knight's tour in chess

Topology & Networks
Knots
Steiner problem
Chinese postman problem
Travelling Salesman Problem
Königsberg bridge problem
Handshake problem
Möbius strip
Klein bottle

Logic & Sets
Codes and ciphers
Set theory and different 'size' infinities
Mathematical induction (strong)
Proof by contradiction
Proving that a number is irrational

Numerical Analysis
Linear programming
Fixed point iteration
Methods of approximating π
Applications of iteration
Newton's method
Estimating size of large crowds
Generating the number e
Descartes' rule of signs
Methods for solving differential equations

Physical, Biological & Social Sciences
Radiocarbon dating
Gravity, orbits & escape velocity
Mathematical methods in economics
Biostatistics
Genetics
Crystallography
Computing centres of mass
Elliptical orbits
Logarithmic scales – decibel, Richter, etc
Fibonacci sequence and spirals in nature
Predicting an eclipse
Change in BMI for a person over time
Concepts of equilibrium in economics

Miscellaneous
Paper folding
Designing bridges
Methods of approximating π
Mathematical card tricks
Curry's paradox – 'missing' square
Barcodes
Applications of parabolas
Music – notes, pitches, scales, etc
Voting systems
Flatland by Edwin Abbott (book)
Terminal velocity
Towers of Hanoi puzzle
Photography
Art of M.C. Escher
Harmonic mean
Sundials
Navigational systems
A Beautiful Mind (film)
The abacus
Construction of calendars
Slide rules
Different number systems
Mathematics of juggling
Global positioning system (GPS)
Airline routes

Mathematics HL Internal Assessment ☑ Student Checklist

Student Name: _____ Date: _____

1. Is your IA written entirely by yourself, and trying to avoid simply replicating work and ideas from sources you found during research? ☐ Yes ☐ No

2. Have you strived to apply your personal interest, develop your own ideas, and use critical thinking skills during your exploration and demonstrate these in your report? ☐ Yes ☐ No

3. Have you referred to the five assessment criteria while writing your report? ☐ Yes ☐ No

4. Does your report focus on good mathematical communication, and read like an article for a mathematical journal? ☐ Yes ☐ No

5. Have you included a title page in proper IB format, as defined by your teacher? ☐ Yes ☐ No

6. Have you used 1 inch margins, 1½-spacing, and a 12-point font (Arial or Times New Roman)? ☐ Yes ☐ No

7. Does your report have a clearly identified introduction and conclusion? ☐ Yes ☐ No

8. Have you documented all of your source material in a detailed bibliography, using proper MLA format, in line with the IB academic honesty policy? ☐ Yes ☐ No

9. Not including the bibliography, title page, and any contents listing, is your report 8 to 14 pages? ☐ Yes ☐ No

10. Are graphs, tables and diagrams sufficiently described and labeled? ☐ Yes ☐ No

11. To the best of your knowledge, have you used and demonstrated mathematics that is at the same level, or above, of that studied in IB Mathematics HL? ☐ Yes ☐ No

12. Have you attempted to discuss mathematical ideas, and use mathematics, with a sufficient level of knowledge, understanding, sophistication and rigor? ☐ Yes ☐ No

13. Are formulae, graphs, tables and diagrams in the main body of text? (preferably no full-page graphs and no separate appendices) ☐ Yes ☐ No

14. Have you used technology – such as a GDC, spreadsheet, mathematics software, drawing & word-processing software – to enhance mathematical communication? ☐ Yes ☐ No

15. Have you used appropriate mathematical language (notation, symbols, terminology) and defined key terms? ☐ Yes ☐ No

16. Is the mathematics in your report performed precisely and accurately? ☐ Yes ☐ No

17. Has proper calculator/computer notation and terminology been used? ☐ Yes ☐ No
($y = x^2$ not $y = x \wedge 2$; $\approx$ not $=$ for approx. values ; π not pi; $|x|$ not abs(x); etc)

18. At suitable places in your report – especially in the conclusion – have you included reflective and explanatory comments about the mathematical topic being explored? ☐ Yes ☐ No

Criterion A: Presentation

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration has some coherence or some organization.
2	The exploration has some coherence and shows some organization.
3	The exploration is coherent and well organized.
4	The exploration is coherent, well organized, and concise.

The "presentation" criterion assesses the organization and coherence of the exploration.

A **coherent** exploration is logically developed, easy to follow and meets its aim. This refers to the overall structure or framework, including introduction, body, conclusion and how well the different parts link to each other.

A **well-organized** exploration includes an introduction, describes the aim of the exploration and has a conclusion. Relevant graphs, tables and diagrams should accompany the work in the appropriate place and not be attached as appendices to the document. Appendices should be used to include information on large data sets, additional graphs, diagrams and tables.

A **concise** exploration does not show irrelevant or unnecessary repetitive calculations, graphs or descriptions.

The use of technology is not required but encouraged where appropriate. However, the use of analytic approaches rather than technological ones does not necessarily mean lack of conciseness, and should not be penalized. This does not mean that repetitive calculations are condoned.

Your exploration

To get a good mark for Criterion A: Communication

- ✓ A well organised exploration should have
  - An **introduction** in which you should discuss the context of the exploration
  - A **rationale** which should include an explanation of why you chose this topic
  - A description of the **aim** of the exploration which should be clearly identifiable
  - A **conclusion**.
- ✓ A coherent exploration is logically developed and easy to follow.
- ✓ Your exploration should "read well".
- ✓ Any graphs, tables and diagrams that you use should accompany the work in the appropriate place and not be attached as appendices to the document.
- ✓ A concise exploration is one that focuses on the aim and avoids irrelevancies.
- ✓ A complete exploration is one in which all steps are clearly explained without detracting from its conciseness.
- ✓ It is essential that references are cited where appropriate, i.e.,
  - Your exploration should contain footnotes as appropriate. For example, if you are using a quote from a publication, a formula from a mathematics book, etc, put the source of the quote in a footnote.
  - Your exploration should contain a bibliography as appropriate. This can be in an appendix at the end. List any books you use, any websites you consult, etc.

ж Further Guidance ж

- **Organization** refers to the overall structure or framework, including the introduction, body, conclusion etc.
- **A coherent** exploration displays a logical development and is not difficult to follow ('reads well').
- **A concise** exploration remains focused on the overall aim and avoids irrelevant material.
- Key ideas and concepts need to be clearly explained.
- Graphs, tables and diagrams should be embedded in the text where most appropriate and not be put in an appendix at the end of the document.
- The use of technology is not required but strongly encouraged where appropriate.
- It is absolutely critical that the use of a source is cited (footnoted) at the location where it is used.
- Your bibliography must list all sources (books, websites, etc) you consulted when writing your Exploration.

Criterion B: Mathematical communication

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration contains some relevant mathematical communication which is partially appropriate.
2	The exploration contains some relevant appropriate mathematical communication.
3	The mathematical communication is relevant, appropriate and is mostly consistent.
4	The mathematical communication is relevant, appropriate and consistent throughout.

The "mathematical communication" criterion assesses to what extent the student has:

- used appropriate mathematical language (**notation, symbols, terminology**). Calculator and computer notation is acceptable only if it is software generated. Otherwise it is expected that students use appropriate mathematical notation in their work
- defined **key terms** and variables, where required
- used **multiple forms of mathematical representation**, such as formulae, diagrams, tables, charts, graphs and models, where appropriate
- used a **deductive method** and set out proofs logically where appropriate

Examples of level 1 can include graphs not being labelled, consistent use of computer notation with no other forms of correct mathematical communication.

Level 4 can be achieved by using only one form of mathematical representation as long as this is appropriate to the topic being explored. For level 4, any *minor* errors that do not impair clear communication should not be penalizsed.

Your exploration

To get a good mark for Criterion B: Mathematical presentation

- ✓ You are expected to use correct mathematical notation and terminology when communicating mathematical ideas, reasoning and findings.
- ✓ You are encouraged to choose and use appropriate ICT tools such as graphic display calculators, mathematical software, spreadsheets, databases, drawing and word-processing software, as appropriate, to enhance mathematical communication.
- ✓ You should define key terms, where required.
- ✓ You should express your results to an appropriate degree of accuracy, when applicable.
- ✓ You should always include scales and labels if you use a graph. Tables should have appropriate headings.
- ✓ Variables should be explicitly defined.
- ✓ Do not use calculator or computer notation. For example, use 2^x and not 2^x; use × not *; use 0.028 and not 2.8E-2.

ж Further Guidance ж

▪ You are expected to use mathematical language (notation, symbols & terminology) when communicating mathematical ideas, reasoning and findings.

▪ You should use appropriate technology such as graphic display calculators; and software such as equation editors, spreadsheets, dynamic geometry, computer algebra, drawing and word-processing software along with other mathematical software to enhance the presentation of mathematics in your Exploration.

▪ The meaning of key terms should be clear and any variables or parameters should be explicitly defined.

▪ All graphs, tables & diagrams should be clearly labeled – and include captions where appropriate.

▪ Do not use calculator or computer notation unless it is software generated and cannot be changed.

Criterion C: Personal engagement

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of some personal engagement.
2	There is evidence of significant personal engagement.
3	There is evidence of outstanding personal engagement.

The "personal engagement" criterion assesses the extent to which the student engages with the topic by exploring the mathematics and making it their own. It is not a measure of effort.

Personal engagement may be recognized in different ways. These include thinking independently or creatively, presenting mathematical ideas in their own way, exploring the topic from different perspectives, making and testing predictions. Further (but not exhaustive) examples of personal engagement at different levels are given in the teacher support material (TSM).

There must be evidence of personal engagement demonstrated in the student's work. It is not sufficient that a teacher comments that a student was highly engaged.

Textbook style explorations or reproduction of readily available mathematics without the candidate's own perspective are unlikely to achieve the higher levels.

Significant: The student demonstrates authentic personal engagement in the exploration on a few occasions and it is evident that these drive the exploration forward and help the reader to better understand the writer's intentions.

Outstanding: The student demonstrates authentic personal engagement in the exploration in numerous instances and they are of a high quality. It is evident that these drive the exploration forward in a creative way. It leaves the impression that the student has developed, through their approach, a complete understanding of the context of the exploration topic and the reader better understands the writer's intentions.

Your exploration

To get a good mark for Criterion C: Personal engagement

- ✓ You should choose a topic for your exploration that you are interested in as it will be easier to display personal engagement.
- ✓ You can demonstrate personal engagement by using some of the following different attributes and skills.
  - Thinking and working independently
  - Thinking creatively
  - Addressing your personal interests
  - Presenting mathematical ideas in your own way
  - Asking questions, making conjectures and investigating mathematical ideas
  - Looking for and creating mathematical models for real-world situations
  - Considering historical and global perspectives
  - Exploring unfamiliar mathematics.

ж Further Guidance ж

▪ It is important to choose a topic in which you are genuinely interested.

▪ If it is necessary to include mathematical work from a source such as a textbook in your Exploration then you should endeavor to insert your own comments and description of the work as much as possible.

▪ Ways to show personal engagement include: investigating your own questions & conjectures; making up your own examples; presenting ideas & results in your own words; creating your own models or functions.

Criterion D: Reflection

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of limited reflection.
2	There is evidence of meaningful reflection.
3	There is substantial evidence of critical reflection.

The "reflection" criterion assesses how the student reviews, analyses and evaluates the exploration. Although reflection may be seen in the conclusion to the exploration, it may also be found throughout the exploration.

Simply describing results represents **limited reflection**. Further consideration is required to achieve the higher levels.

Some ways of showing **meaningful reflection** are: linking to the aims of the exploration, commenting on what they have learned, considering some limitation or comparing different mathematical approaches.

Critical reflection is reflection that is crucial, deciding or deeply insightful. It will often develop the exploration by addressing the mathematical results and their impact on the student's understanding of the topic. Some ways of showing critical reflection are: considering what next, discussing implications of results, discussing strengths and weaknesses of approaches, and considering different perspectives.

Substantial evidence means that the critical reflection is present throughout the exploration. If it appears at the end of the exploration it must be of high quality and demonstrate how it developed the exploration in order to achieve a level 3.

Further (but not exhaustive) examples of reflection at different levels are given in the teacher support material (TSM).

Your exploration

To get a good mark for Criterion D: Reflection

- ✓ Although reflection may be seen in the conclusion to the exploration, it may also be found throughout the exploration.
- ✓ You can show reflection in your exploration by
  - Discussing the implications of your results
  - Considering the significance of your findings and results
  - Stating possible limitations and/or extensions to your results
  - Making links to different fields and/or areas of mathematics.

ж Further Guidance ж

▪ Simply describing results represents **limited or superficial reflection**. To achieve a score higher than 1 you will need to provide deeper and more sophisticated consideration of methods and results.

▪ Ways of showing **meaningful reflection** include: linking results to the aim of your Exploration; commenting on what you have learned; considering limitations; or comparing different mathematical approaches.

▪ Ways of showing **critical reflection** include: considering implications of results; discussing strengths and weaknesses of methods; considering different perspectives; making links between different areas of math.

▪ **Substantial evidence** is likely to mean that reflection is present throughout the exploration.

Criterion E: Use of mathematics—HL

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	Some relevant mathematics is used. Limited understanding is demonstrated.
2	Some relevant mathematics is used. The mathematics explored is partially correct. Some knowledge and understanding is demonstrated.
3	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Some knowledge and understanding are demonstrated.
4	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Good knowledge and understanding are demonstrated.
5	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct and demonstrates sophistication or rigour. Thorough knowledge and understanding are demonstrated.
6	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is precise and demonstrates sophistication and rigour. Thorough knowledge and understanding are demonstrated.

The "Use of mathematics" HL criterion assesses to what extent students use **relevant** mathematics in the exploration.

Students are expected to produce work that is **commensurate with the level** of the course, which means it should not be completely based on mathematics listed in the prior learning. The mathematics explored should either be part of the syllabus, at a similar level or slightly beyond. However, mathematics of a level slightly beyond the syllabus is **not** required to achieve the highest levels.

A key word in the descriptor is **demonstrated**. The command term demonstrate means to make clear by reasoning or evidence, illustrating with examples or practical application. Obtaining the correct answer is not sufficient to demonstrate understanding (even some understanding) in order to achieve level 2 or higher.

For knowledge and understanding to be **thorough** it must be demonstrated throughout. Lines of reasoning must be shown to justify steps in the mathematical development of the exploration.

Relevant refers to mathematics that supports the development of the exploration towards the completion of its aim. Overly complicated mathematics where simple mathematics would suffice is not relevant.

The mathematics can be regarded as **correct** even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome. **Precise** mathematics is error-free and uses an appropriate level of accuracy at all times.

Sophistication: To be considered as sophisticated the mathematics used should be commensurate with the HL syllabus or, if contained in the SL syllabus, the mathematics has been used in a complex way that is beyond what could reasonably be expected of an SL student. Sophistication in mathematics may include understanding and using challenging mathematical concepts, looking at a problem from different perspectives and seeing underlying structures to link different areas of mathematics.

Rigour involves clarity of logic and language when making mathematical arguments and calculations. Mathematical claims relevant to the development of the exploration must be justified or proven.

Students are encouraged to use technology to obtain results where appropriate, but **understanding must be demonstrated** in order for the student to achieve level 1 or higher, for example merely substituting values into a formula does not necessarily demonstrate understanding of the results.

The mathematics only needs to be what is required to support the development of the exploration. This could be a few small elements of mathematics or even a single topic (or sub-topic) from the syllabus. It is better to do a few things well than a lot of things not so well. If the mathematics used is relevant to the topic being explored, commensurate with the level of the course and understood by the student, then it can achieve a high level in this criterion.

Your exploration

To get a good mark for Criterion E: Use of mathematics

- ✓ You are expected to produce work that is commensurate with the level of the course you are studying. The mathematics you explore should either be part of the syllabus, or at a similar level (or beyond).
- ✓ You should ensure that the mathematics involved is not completely based on mathematics listed in the prior learning.
- ✓ **If the level of mathematics is not commensurate with the level of the course you can only get a maximum of two marks for this criterion.**
- ✓ You need to demonstrate within your exploration that you fully understand the mathematics used.
- ✓ You can demonstrate sophistication of mathematics in your exploration by
  - Showing that you understand and can use challenging mathematical concepts
  - Showing that you can extend the applications of mathematics beyond that which you learned in the classroom
  - Looking at a problem from different mathematical perspectives
  - Identifying underlying structures to link different areas of mathematics.
- ✓ Rigor involves clarity of logic and language when making mathematical arguments and calculations.
- ✓ Precise mathematics is error-free and uses an appropriate level of accuracy at all times.

ж Further Guidance ж

- It is critical that you clearly demonstrate that you understand the mathematical concepts and methods that you write about in your Exploration.
- **Sophistication in mathematics** may include understanding & use of challenging math concepts, looking at a problem from different perspectives and seeing underlying structures to link different areas of mathematics.
- **Rigour** involves clarity of logic and language when making mathematical arguments and calculations.
- **Precise mathematics** is error-free and uses an appropriate level of accuracy at all times.