Mathematical Investigation

Starting a Blood Bank



Introduction

Blood transfusions may be necessary in case of surgery, injury, bleeding, cancer, infections, blood diseases, and more. In the case of a transfusion, another person donates blood to administer to the patient. However, not any two people can share blood. There are four different blood groups, and not all are compatible with each other. Blood banks in hospitals are in charge of stocking and sorting blood samples, and it's vital that a sufficient amount of blood is available to suit every patient.

The distribution of blood types within a population varies for each country and ethnic group around the world. This investigation only uses blood group distribution for the United States. The U.S. has precise statistical data, ethnic diversity, and a large population sample.

This investigation explores how many units of blood need to be collected within the first month of opening a blood bank to have a sufficient amount of samples. Here "sufficient" is defined to be a 99.9% chance of not running out of blood during a one-month period. It's important that the question considers the first month, because the stock of blood needs to be built up when starting with 0 samples. I expect the results to be in the range of a few hundreds of samples per month. Anything over that number would be unrealistic for the average hospital. However it must be noted that this situation wouldn't occur in real life. If a new blood center opened, it would receive the necessary blood samples from a provider such as the Red Cross.

Research Question

When opening a blood bank in the United States, how many random people are needed to donate blood within the first month to have a safe amount of units to address the population's needs in most of the cases taking into account the predominant ethnicities?

Rationale

I chose the topic of blood groups because I could see there are a wide variety of mathematics relating to it. I am interested in the topic because it is relevant to the world nowadays. I'm also interested in biology in general and enjoy studying it. Our school organizes blood drives for the Red Cross twice a year, which gave me the idea of researching blood transfusions and statistics. The topic of blood transfusions is relevant because it is a process that saves thousands of lives every day. Clear organization is needed to run blood centers and to make the transfusion process as efficient as possible.

When researching the topic I noticed there were few investigations that asked the same question as mine, and I wanted my investigation to be original. However this prevents me from comparing my results to similar studies, and it is up to me to evaluate whether they are logical.

Assumptions

This investigation assumes that the blood stored in blood centers is representative of the whole population. Since no blood group is more prone to be refused at blood banks, there should be a similar proportion of blood types within a blood bank as within the U.S. Additionally, it's assumed that the proportions of donors' ethnicities match those of the country. Another assumption is that every hospital is considered as one blood center. This investigation also considers one unit of blood as one pint, although the two volumes are very close (450 mL and 473 mL respectively).

Background Information

The Blood Groups

There are four main blood types for humans, classified using the ABO system. The four groups are A, B, AB, and O. Blood type is determined by the presence or absence of specific antigens. Blood is additionally classified as positive or negative, depending on the presence of Rhesus (Rh) factors. Blood is Rh-positive if it contains Rh factors and conversely ("Blood Types").

This further classification results in a total of eight different blood types in humans. However, because this distinction isn't necessarily vital, this experiment will not take Rh factors into account to simplify calculations.

Transfusions

Antigens are substances that can trigger an immune response if they are foreign to the body. For this reason it's important to transfer the correct blood type to a patient. Receiving an incompatible blood type leads to a blood transfusion reaction. This is caused by the patient's antibodies attacking the foreign blood cells, leading to hemolysis (the bursting of the cells). This kind of reaction can lead to severe complications and possibly death. Nowadays, blood types are carefully monitored and transfusion reactions are very rare ("Transfusion Reaction").

All blood types have specific compatibilities for transfusions. Each can donate to and receive from different groups. Naturally, people can exchange blood with people of the same blood type. However, additional combinations are possible among different ones. The restrictions arise from the interaction between each type's antigens and antibodies. Anti-A antibodies will attack blood containing the A antigen, and Anti-B antibodies will attack blood containing the B antigen. As a result, AB blood can only donate to AB, because it contains both antigens but no antibodies. However its lack of antibodies allows it to receive transfusions from any other blood type. On the contrary, O blood can donate to every other blood type because it contains neither antigen. But because it contains both antibodies, it can only receive O blood ("ABO Blood Types").

The Rh compatibility is simpler. People with Rh-positive blood can receive transfusions from either Rh-positive or negative blood. However people with Rh-negative blood should only get Rh-negative red blood cells, except in extreme emergencies ("Getting a Blood Transfusion."). For the purpose of this experiment, it is assumed that blood transfusions are conducted regardless of the Rh factors.



This diagram (Fig. 1) summarizes the compatibility between the four main blood groups.

(Fig. 1) Source: "Blood Types." Redcrossblood.org. Red Cross, n.d. Web. 20 Dec. 2016.

More discrete antigens affect transfusions between patients of different ethnicities. For example, U-negative and Duffy-negative blood types are unique to the African-American community. These discrete antigens can cause transfusion reactions and cause some complications when transferred to a patient of a different ethnicity. To avoid this risk, transfusions are performed between patients of the same ethnicity ("Blood and Diversity").

Data and Statistics

Different ethnic groups have different distributions of blood types. The data in Figure 2 was collected by the Red Cross, and it shows the mix of blood types of the four main ethnic groups in the United States.

	Caucasian	African- American	Latino-American	Asian
0+	37%	47%	53%	39%
O -	8%	4%	4%	1%
A +	33%	24%	29%	27%
A -	7%	2%	2%	0.5%
в+	9%	18%	9%	25%
в-	2%	1%	1%	0.4%
AB +	3%	4%	2%	7%
AB -	1%	0.3%	0.2%	0.1%

(Fig. 2) Source: "Blood Types." Redcrossblood.org. Red Cross, n.d. Web. 20 Dec. 2016.

Because this investigation doesn't take into account Rhesus factors, Figure 3 combines the data into the 4 main blood types. These are the numbers used in this investigation.

%	Caucasian	African-American	Latino-American	Asian
0	45	51	57	40
Α	40	26	31	27.5
В	11	19	10	25.4
AB	4	4.3	2.2	7.1

(Fig. 3) Cumulative distribution of blood types in different ethnic groups

Below is the distribution by race and ethnicity in the U.S. population in 2015, according to the United States Census Bureau (Fig. 4).

	Caucasian	African-American	Latino-American	Asian	Other
%	61.6	13.3	17.6	5.6	1.9

(Fig. 4) Source: United States Census Bureau: QuickFacts, 2015. Web. Accessed 19 Feb. 2017.

Blood Banks

Blood donors aren't denied based on their blood type. Donations are only refused for reasons that compromise the blood such as a disease or intravenous drugs, etc. Since no single blood group is significantly more likely to be rejected from a blood bank, donations should mirror the population.

There were 5,564 registered hospitals in the United States as of 2015. This number comprises all "general" hospitals accessible to the public, and not individually described specialty services such as obstetrics and gynecology; eye, ear, nose, and throat; rehabilitation; orthopedic, etc. ("Fast Facts on US Hospitals"). This investigation assumes that there is one blood bank per hospital. This new hypothetical blood bank would then be the 5,565th.

The following statistics are directly used in the calculations of this investigation. They were all provided by the American Red Cross. Approximately 36,000 units of red blood cells are needed every day in the U.S. The average transfusion patient receives 3 units of red blood cells. About 1 pint of blood (one unit) is given during a donation ("Blood Facts and Statistics").

Calculations

In many of the following calculations, results only make sense as whole numbers (for example a number of people). However calculations were followed through with decimals, to ensure a more accurate final answer. Answers are either rounded to the nearest whole number or the integer above, depending on the case. Since 36,000 units of blood are needed each day in the United States and there are 5,564 hospitals, then the mean number of units needed per month in this 5,565th blood bank is

$$\mu = (36000 \times 30) / 5565 = 194.1$$
 units.

This mean number of units per month follows a Poisson. I chose the probability of failure to be 0.001 and the chance of success 0.999, meaning there is a 0.1% chance of running out of blood samples. Calculating the inverse Poisson distribution using these parameters finds the area (α) under 99.9% of the curve. This is the number of units needed to be collected the first month so as to have a safety margin and not run out of blood in 99.9% of cases.

$$\alpha = 1 - 0.001 = 0.999$$

$$\mu = 194.1 \text{ units needed/month}$$

$$X = inversePoisson(0.999, 194.1)$$

$$X = 239 \text{ units needed}$$

Subtracting the mean number of units to be collected from the total number of units needed in stock results in the margin of safety, which acts like a buffer. An abnormal consumption of more than 194 units per month is possible. Even if more than 194 units are used, there will still be some available in the safety stock.

239-194 = 45 units as margin of safety

Since there is a 0.1% chance of failure there will be a failure every 1000 months, which means there will be a shortage of blood every 83 years and 4 months.

$$0.001 = \frac{1}{1000}$$
$$1000 / 12 = 83.\overline{3} \text{ years}$$

The average blood transfusion requires 3 units of blood, so about 65 patients can be treated with 194.1 units of blood.

$194.1/3 = 64.7 \approx 65$ patients/month

The following calculations are based on patients of blood group B, which I selected by elimination. AB is the least common blood group, but as the universal donor it can receive from any donor thus eliminating the restriction of blood types. Group O is the most common and can only receive O blood, but using it in the following calculations consistently results in the original value of 194.1 units. I chose group B because it is less common in all four ethnicities than group A. Therefore group B provides the best balance between rarity in donors and patients. If the least common blood group is available in a blood bank, then the other types of blood should be too. The first ethnicity calculated is the Asian population, which composes 5.6% of the total U.S. population. The B blood group is present in 25.4% of the Asian population.

$$64.7 \times 0.056 = 3.62 \approx 4$$
 Asian patients/month
 $3.62 \times 0.254 = 0.919$ B Asians/month
 $0.919 \times 3 = 2.76$ units needed

Out of 65 patients, 4 are Asian. Out of those four, 0.919 have type B blood. Because on average 3 units of blood are required for a transfusion, 2.76 units of blood are needed to cover all transfusions for Asian patients of type B. Both type O and B blood are suitable for this transfusion, however it is best if the donor is Asian too.

To find the probability of a person having O or B blood given they are Asian, conditional probability is used.

$$P(A|B) = \frac{P(A \bigcap B)}{P(B)} \rightarrow P(A \bigcap B) = P(A|B) \times P(B)$$
$$P(O \text{ or } B \bigcap \text{Asian}) = (0.40 + 0.254)(0.056) = 0.0366$$

Of the U.S. population, 3.66% are Asian and have type O or B blood. The next step is to find the number of random donors needed to obtain the 2.76 units of blood needed. Binomial distribution

is used for this step, because the donor either meets these conditions or doesn't. The expected value E(X) for a binomial distribution equals the number of trials multiplied by the probability of success. In this case we are looking for *n* donors. The number is rounded above in order to meet the demand.

$$E(X) = np$$

2.54 = n(0.0366)
$$n = 75.4$$

$$n \approx 76 \text{ people}$$

The number of random donors needed to suit the needs of all Asian patients is 76.

This process is then repeated for the other three ethnicities studied.

African-American people make up 13.3% of the U.S. population, and 19.0% of that population has blood type B.

 $64.7 \times 0.133 = 8.61 \approx 9$ African-American patients/month $8.61 \times 0.19 = 1.64$ B African-Americans/month $1.64 \times 3 = 4.91$ units needed $P(O \text{ or } B \bigcap \text{African-American}) = (0.51 + 0.19)(0.133) = 0.0931$ E(X) = np 4.91 = n(0.0931) n = 52.7 $n \approx 53$ people

Latino-American people make up 17.6% of the U.S. population, and 10.0% of that population has blood type B.

$$64.7 \times 0.176 = 11.4 \approx 11$$
 Latino patients/month
 $11.4 \times 0.10 = 1.14$ B Latinos/month
 $1.14 \times 3 = 3.42$ units needed
 $P(O \text{ or } B \bigcap \text{Latino}) = (0.57 + 0.10)(0.176) = 0.118$
 $E(X) = np$
 $3.42 = n(0.118)$
 $n = 29.0$
 $n \approx 29$ people

Caucasian people make up 61.6% of the U.S. population, and 11.0% of that population has blood type B. $64.7 \times 0.616 = 39.9 \approx 40$ Caucasian patients/month

$$39.9 \times 0.11 = 4.38$$
 B Caucasian patients/month
 $4.38 \times 3 = 13.2$ units needed
 $P(O \text{ or } B \bigcap \text{Caucasian}) = (0.45 + 0.11)(0.616) = 0.345$
 $E(X) = np$
 $13.2 = n(0.345)$
 $n = 38.1$
 $n \approx 39$ people

Then, the numbers of donors needed for each ethnicity are added together.

$$76+53+29+39=197$$
 donors needed

If 197 random people donated blood to the blood bank each month, it would guarantee having the correct type of blood for at least 98.1% of people within the United States. This would be the minimum number of donations needed to start a safe blood bank. However, since 239 are needed the first month, the chance of covering the needs of the entire population is very close to 100%.

Since only 3 samples of blood are needed each month for Asian B patients (rounded from 2.76), and that there are 239 units available, the probability of not having an appropriate sample is equal to

$$P(X) = \frac{n!}{X!(n-X)!} * (p)^{X} * (q)^{n-X}$$

$$n = 239$$

$$X = 0$$

$$p = 0.0366$$

$$q = 0.9634$$

$$BinomPdf (239, 0.0366, 0) = 0.000135 = 0.0135\%.$$

Conclusion

Evaluation of results

The number of donations needed during the first month of a blood bank is 239 units, considering that an average of 194 units are used per month. This results in a margin of safety of 45 units. These numbers result from allowing a 0.1% probability of depleting the stock. This safety stock would then be depleted once every 1000 months, which is every 83 years.

To ensure that the correct samples of blood are available to treat almost any patient, 197 random people would be needed to donate blood, since they can each donate one unit. It's logical that more than the minimum 194 samples are needed. If the calculated 239 people donate, it's 99.99% sure that the appropriate samples will be available to treat patients.

These results seem reasonable. They indicate that about 6 to 8 people need to donate each day, which is plausible. Furthermore these calculations evaluate the less likely blood types for patients. Realistically many patients would have the more common O and A blood types.

My results meet the aim of my investigation. Although this situation is hypothetical, it relates to the importance of having the appropriate blood samples to treat any patient. I was able to find useful numbers relevant to blood banks and the organization of transfusions. Additionally my hypothesis was correct.

Limitations and challenges faced

This investigation heavily relies on statistics, but these are quickly-changing. I tried to use data from the same year, usually 2015, but the Red Cross data used in the calculations may be taken from a different year.

I originally conducted all of my procedure with blood type O, but I realized it did not work as my calculations for each ethnicity would loop back to the original value of 194.1. I had chosen O because it is the most highly demanded in hospitals, due to its property of universal donor, and because type O patients can only receive transfusions from the same blood group.

I wanted to find statistics about ethnic distribution in the four main regions of the U.S. I intended to use this data to use in conditional probability with distribution of blood groups within ethnicities, to make the results more precise according to the region. However I could not find any suitable statistics; only percentages of specific countries of origin or imprecise maps.

To improve this investigation, the calculations could be made for a more specific region, using population data. Here, the number of samples needed is calculated for the average U.S. hospital. It doesn't take into account that some areas are more densely populated than others and have different proportions of ethnicities. Overall, more regionally precise data could be used instead of national statistics. Taking into account Rhesus factors and other discrete antigens would also make this investigation much more precise; I simplified mine by only using the four main blood groups.

Works Cited

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Assessment criteria:

Criterion		Comments *	Achievement level		evel
			Teacher	Moderator	Senior moderator
A	Communication	Exploration is well-organized, contains a very focused theme, and all relevant information is presented. Key sources are cited, and a clear explanation was given regarding blood types to support goal.	0-4		
в	Mathematical presentation	Tables were clearly labeled and on topic, equations were written well in proper formats, and the variety of info, graph, tables, and equations supported the overall theme of the paper.	0-3		
с	Personal engagement	Although the topic was unique and interesting, and the personal interest was apparent, it lacked depth. Connections to actual blood bank needs was good, but deeper questions were needed.	0-4		
D	Reflection	This section is where the student excelled. He identified many possible extensions and limitations of his paper, and discussed the meaningful results and relevance to real-world situations.	0-3		
E	Use of mathematics	Although the mathematics used was appropriate and correct, they lacked a level of difficulty and rigor. The use of Poisson distribution in this situation was the best choice, but maybe not a good one.	0-6 3		
		Total:	0-20		