① $f(x) = e^{-x}\cos x + x - 1$

$e^{x} = 1 + x + \frac{x^2}{2!} + \dots$

$e^{-x} = 1 - x + \frac{x^2}{2!} - \dots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

$e^{-x}\cdot\cos x = \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}\right)\left(1 - \frac{x^2}{2!} + \cancel{\frac{x^4}{4!}} - \dots\right)$

$= \cancel{1} - \cancel{\frac{x^2}{2!}} - \cancel{x} \boxed{+ \frac{x^3}{2!}} + \cancel{\frac{x^2}{2!}} - \frac{x^3}{3!}$

$e^{-x}\cdot\cos x + x - 1 = \frac{1}{2}x^3 - \frac{1}{6}x^3$

$= \boxed{\frac{1}{3}x^3}$

② $\left(x\cdot\frac{dy}{dx} + y = f(x)\right)\cdot\frac{1}{x}$

a.) $\frac{dy}{dx} + \frac{1}{x}\cdot y = \frac{1}{x}\cdot f(x)$

$I(x) = e^{\int \frac{1}{x}\cdot dx}$

$I(x) = e^{\ln x}$

$I(x) = x$

WAS ALREADY OK

$x\cdot\frac{dy}{dx} + y = f(x)$

$xy = \int f(x) + c$

$y = \frac{1}{x}\int f(x)\cdot dx$

$xy = \int f(x)\cdot dx$

b.) $x\cdot\frac{dy}{dx} + y = x^{-1/2}$

$xy = 2x^{1/2} + c$ $(4,2)$

$4\cdot 2 = 2\sqrt{4} + c$

$8 = 4 + c$

$c = 4$

$xy = 2\sqrt{x} + 4$

$y = \frac{2\sqrt{x}}{x} + \frac{4}{x}$

$\boxed{y = \frac{2\sqrt{x} + 4}{x} \text{ or } y = \frac{2}{\sqrt{x}} + \frac{4}{x}}$

3) a.) $\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n} = \sum_{n=2}^{3} \frac{1}{n^2 \ln n} + \sum_{n=3}^{\infty} \frac{1}{n^2 \ln n}$

$= \frac{1}{4 \ln 2} + \frac{1}{9 \ln 3} + \sum_{n=3}^{\infty} \frac{1}{n^2 \ln n}$

$\frac{1}{n^2 \ln n} < \underbrace{\frac{1}{n^2}}$

$\frac{1}{n^2}$ CONV. $\left(\frac{1}{n^p}, p > 1\right)$ P-SERIES

$\frac{1}{n^2 \ln n}$ CONV BY COMP. TEST

$\therefore \sum_{n=2}^{\infty} \frac{1}{n^2 \ln n}$ CONVERGES.

b.) $\ln(n) + \ln\left(1 + \frac{1}{n}\right) = \ln(n+1)$

$\ln\left(n\left(1 + \frac{1}{n}\right)\right) = \ln(n+1)$

$\ln(n+1) = \ln(n+1)$ ✓

NO $\lim_{b \to \infty}$ FOR TIME SAKE

c. $\int_2^{\infty} \frac{1}{n \ln n} \, dn$ $\quad u = \ln n$, $du = \frac{1}{n} dn$ $\quad \int_2^{\infty} \frac{1}{u} du = [\ln|u|]_2^{\infty} = [\ln|\ln n|]_2^{\infty}$

$= \ln|\ln \infty| - \ln|\ln 2|$

$= \ln(\ln \infty - \ln 2)$

$= \ln\left(\ln\left(\frac{\infty}{2}\right)\right)$

$= \infty$

DIVERGES

④ a.) $\lim_{x\to\infty} x^2e^{-x} = \lim_{x\to\infty} \frac{x^2}{e^x}\left(\frac{\infty}{\infty}\right) = \lim_{x\to\infty}\frac{2x}{e^x}\left(\frac{\infty}{\infty}\right) = \lim_{x\to\infty}\frac{2}{e^x} = \boxed{0}$

b.) $\int_0^\infty x^2e^{-x}\,dx = -e^{-x}x^2 + \int 2xe^{-x}\,dx = -e^{-x}x^2 - 2xe^{-x} + 2\int e^{-x}\,dx$

$u = x^2 \quad dv = e^{-x}\,dx$

$du = 2x\,dx \quad v = -e^{-x}$

$u = 2x \quad dv = e^{-x}dx$

$du = 2dx \quad v = -e^{-x}$

$= -e^{-x}x^2 - 2xe^{-x} - 2e^{-x} + C$

$= e^{-x}(-x^2 - 2x - 2)$

$\lim_{b\to\infty}\left[e^{-x}(-x^2-2x-2)\right]_0^b$

$= e^{-b}(-b^2-bx-2) - e^0(0+0-2)$

$= \frac{-b^2-bx-2}{e^b} + 2$

0

$= \boxed{2}$ CONVERGES

$-6 \pm \sqrt{36}$

⑤ a.) $f(-3) = -2$

$f(1) = 2$

$\frac{f(b)-f(a)}{b-a} = \frac{2-(-2)}{1-(-3)} = \frac{4}{4} = 1$

$f'(x) = 3x^2 + 6x$

$3x^2 + 6x = 1$

$3x^2 + 6x - 1 = 0$

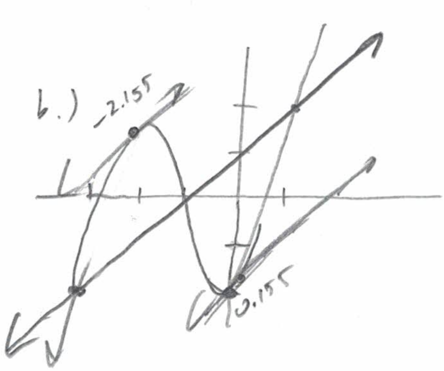
$x = \frac{-6 \pm \sqrt{36 + 4(3)(1)}}{6}$

$x = \frac{-6 \pm \sqrt{48}}{6} = \frac{-6 \pm 4\sqrt{3}}{6}$

$x = \frac{-3 \pm 2\sqrt{3}}{3}$

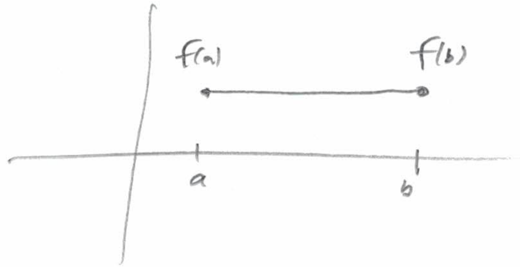
$x = 0.155, -2.155$

b.)



5(b) ↷ over

(5) b.) $f'(x)=0$ Means Horiz. Tang Line.



$$\cos\left(2\arccos x + \arccos(1-2x^2) = \pi\right)$$

$$\cos\left(2\arccos x + \arccos(1-2x^2)\right) = \cos \pi$$

Sum / Difference

$$\cos(\underbrace{2\arccos x}_{2\theta})\cos(\arccos(1-2x^2)) - \sin(\overset{2\theta}{2\arccos x})\sin(1-x^2)$$

$$\left(2\cos^2(\arccos x)-1\right)(1-x^2) - (2\sin(\arccos x)\cos$$

$$f(x) = 2\arccos x + \arccos(1-2x^2)$$

$$f(0) = 2\cdot\frac{\pi}{2} + 0 = \pi$$

$$f(1) = 2\cdot 0 + \pi = \pi$$ ✓

$\therefore\ 2\arccos x + \arccos(1-2x^2) = \pi$

$$f'(x) = \frac{-2}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-(1-2x^2)^2}}\cdot -4x$$

$$= \frac{-2}{\sqrt{1-x^2}} + \frac{4x}{\sqrt{1-(1-4x^2+4x^4)}}$$

$$= \frac{-2}{\sqrt{1-x^2}} + \frac{4x}{\sqrt{4x^2-4x^4}}$$

$$= \frac{-2}{\sqrt{1-x^2}} + \frac{4x}{\sqrt{4x^2(1-x^2)}}$$

$$= \frac{-2}{\sqrt{1-x^2}} + \frac{\cancel{4x}\,2}{\cancel{2x}\sqrt{1-x^2}}$$

$$f'(x) = 0$$ ✓