Math HL – Year 1
Complex Numbers – Review

Name: KEY
Period: ____

Topic List

- Complex Numbers **(#2-10)**
  - (3F) Real vs Imaginary parts
  - (3F) Modulus (Notation |z| and using Pythagorean Theorem to determine distance from (0,0))
- Operations with Complex Numbers **(#2-5,8,10)**
  - (3F) Addition and subtraction (Like terms, real and imaginary)
  - (3G) Multiplication (Simplifying with powers of i)
  - (3G) Division (Complex conjugates)
  - (3G) Solving for the complex number z (using add, subt, mult, and div)
- Higher Powers of i and Complex Roots **(#6-8,10)**
  - (3H) Patterns with powers of i repeating every 4 powers ($i^0 = 1, i^1 = i, i^2 = -1, i^3 = -i$)
  - (3H) Complex Roots (using $a^2 - b^2$ = Real and $2ab$ = Imaginary)
- Synthetic Division and Polynomial Roots (Both real and complex) **(#1,5)**
  - (3J and 3K) Real Roots and Rational Root Theorem **(3K #5,6 are important!)**
  - (3L) Complex Roots and Conjugate Root Theorem (Complex roots exist in conjugate pairs)
- Systems of Equations **(#9)**
  - (3R) Solving 3 Variable Systems

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1. (a) Show that $p = 2$ is a solution to the equation $p^3 + p^2 - 5p - 2 = 0$.

$$\begin{array}{c|cccc} 2 & 1 & 1 & -5 & -2 \\ & \downarrow & 2 & 6 & 2 \\ \hline & 1 & 3 & 1 & \boxed{0} \end{array}$$

(b) Find the values of a and b such that $p^3 + p^2 - 5p - 2 = (p - 2)(p^2 + ap + b)$.

$p^2 + ap + b = p^2 + 3p + 1$

$\boxed{a = 3, \ b = 1}$

(c) Hence find the other two roots to the equation $p^3 + p^2 - 5p - 2 = 0$.

$p^2 + 3p + 1 = 0$

DOES NOT FACTOR,

USE QUADRATIC FORMULA

$p = \dfrac{-3 \pm \sqrt{9 - 4(1)(1)}}{2}$

$\boxed{p = \dfrac{-3 \pm \sqrt{5}}{2}}$

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2. Find the values of a and b, where a and b are real, given that $(a + bi)(2 - i) = 5 - i$.

$$a+bi = \frac{5-i}{2-i}$$

$$a+bi = \frac{(5-i)(2+i)}{(2-i)(2+i)}$$

$$= \frac{10+3i-i^2}{4-i^2}$$

$$= \frac{11+3i}{5}$$

$$\boxed{a+bi = \frac{11}{5} + \frac{3}{5}i}$$

3. Given $(5+zi)(2-i) = 2z+8i$, find the value of z in the form $a + bi$.

$$10-5i+2iz-zi^2 = 2z+8i$$

$$\cancel{10}-\cancel{5i}+2iz+z = \cancel{2z}+8i$$

$$-10+5i \qquad -2z$$

$$-z+2iz = -10+13i$$

$$z(-1+2i) = -10+13i$$

$$z = \frac{-10+13i}{-1+2i}$$

$$z = \frac{(-10+13i)(-1-2i)}{(-1+2i)(-1-2i)}$$

$$z = \frac{10+20i-13i-26i^2}{1-4i^2}$$

$$z = \frac{36+7i}{5}$$

$$\boxed{z = \frac{36}{5} + \frac{7}{5}i}$$

4. Given that $(a + i)(2 - bi) = 7 - i$, find the value of a and of b, where $a, b \in \mathbb{Z}$.

$$2a-abi+2i-bi^2 = 7-i$$

$$\underbrace{2a+b}_{\mathbb{R}} \underbrace{-abi}_{i} = \underbrace{7}_{\mathbb{R}} \underbrace{-3i}_{i}$$

$$2a+b = 7$$

$$-ab = -3$$

$$ab = 3$$

SOLVE FOR EITHER a OR b

$$a = \frac{3}{b}$$

$$2\left(\frac{3}{b}\right)+b = 7$$

$$\left(\frac{6}{b}+b = 7\right)\cdot b$$

$$6+b^2 = 7b$$

$$b^2-7b+6 = 0$$

$$(b-6)(b-1) = 0$$

$$\cancel{b=6} \qquad b=1$$

$$\cancel{a} \qquad a=3$$

$a, b \in \mathbb{Z}$

$$\boxed{a=3,\ b=1}$$

5. Given that $2 + i$ is a root of the equation $x^3 - 6x^2 + 13x - 10 = 0$ find the other two roots.

CONJUGATE ROOT THM

$$\begin{array}{r|rrrr} 2+i & 1 & -6 & 13 & -10 \\ & \downarrow & 2+i & -9-2i & 10 \\ \hline 2-i & 1 & -4+i & 4-2i & \boxed{0} \\ & \downarrow & 2-i & -4+2i & \\ \hline & 1 & -2 & \boxed{0} & \end{array}$$

$(2+i)(-4+i)$

$-8-2i+i^2$

$-9-2i$

$(2+i)(4-2i)$

$8-2i^2$

10

$\boxed{\text{ROOTS: } 2,\ 2 \pm i}$

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6. Find the two square roots of $6-8i$.

$\sqrt{6-8i}$

$a^2-b^2=6$

$2ab=-8$

SOLVE FOR a

$a=\frac{-4}{b}$

$\left(-\frac{4}{b}\right)^2-b^2=6$

$\left(\frac{16}{b^2}-b^2=6\right)\cdot b^2$

$b^4+6b^2-16=0$

$(\cancel{b^2+8})(b^2-2)=0$

$b=\pm\sqrt{2}$

$\downarrow$

$a=\mp 2\sqrt{2}$

$\boxed{\begin{array}{c} 2\sqrt{2}-\sqrt{2}i \\ -2\sqrt{2}+\sqrt{2}i \end{array}}$

SOLVE FOR b

$b=\frac{-4}{a}$

$a^2-\left(-\frac{4}{a}\right)^2=6$

$a^2-\frac{16}{a^2}=6$

$a^4-6a^2-16=0$

$(a^2-8)(\cancel{a^2+2})=0$

$a=\pm 2\sqrt{2}$

$\downarrow$

$b=\mp\sqrt{2}$

$\boxed{\begin{array}{c} 2\sqrt{2}-\sqrt{2}i \\ -2\sqrt{2}+\sqrt{2}i \end{array}}$

ROOTS

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7. Find the two square roots of $5+12i$.

$a^2-b^2=5$

$2ab=12$

SOLVE FOR a

$a=\frac{6}{b}$

$\left(\frac{6}{b}\right)^2-b^2=5$

$\downarrow$

$b^4+5b^2-36=0$

$(\cancel{b^2+9})(b^2-4)=0$

$b=\pm 2 \longrightarrow a=\pm 3$

SOLVE FOR b

$b=\frac{6}{a}$

$a^2-\left(\frac{6}{a}\right)^2=5$

$\downarrow$

$a^4-5a^2-36=0$

$(a^2-9)(\cancel{a^2+4})=0$

$a=\pm 3 \longrightarrow b=\pm 2$

$\boxed{\begin{array}{c} \text{ROOTS} \\ \hline 3+2i \\ -3-2i \end{array}}$

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8. Expand $(2-3i)^4$, expressing your answer in the form $a+bi$.

$(2-3i)^2=(2-3i)(2-3i)$

$=4-12i+9i^2$

$=\underline{-5-12i}$

$(-5-12i)^2=(-5-12i)(-5-12i)$

$=25+120i+144i^2$

$\boxed{-119+120i}$

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9. Solve the following 3 variable systems.

(a) ① $2x-3y+z=10$
② $5x+2y+2z=15$
③ $x+4y-2z=-3$

$2\cdot$① $4x-6y+\cancel{2z}=20$
③ $x+4y-\cancel{2z}=-3$
Ⓐ $5x-2y=17$

② $5x+2y+\cancel{2z}=15$
③ $x+4y-\cancel{2z}=-3$
$6x+6y=12$) $\div 2$
Ⓑ $x+y=2$

Ⓐ $5x-\cancel{2y}=17$
$2x+\cancel{2y}=4$
$7x=21$ $\boxed{x=3}$

$3+y=2$
$\boxed{y=-1}$

③ $3-4-2z=-3$
$-2z=-2$
$\boxed{z=1}$

$\boxed{(3,-1,1)}$

(b) ① $2x+14y+9z=-7$
② $4x-7y-3z=4$
③ $10x-28y-6z=5$

① $2x+\cancel{14y}+9z=-7$
$2\cdot$② $8x-\cancel{14y}-6z=8$
Ⓐ $10x+3z=1$

$2\cdot$① $4x+\cancel{28y}+18z=-14$
③ $10x-\cancel{28y}-6z=5$
Ⓑ $14x+12z=-9$

$-4\cdot$Ⓐ $-40x-\cancel{12z}=-4$
Ⓑ $14x+\cancel{12z}=-9$
$-26x=-13$
$\boxed{x=\frac{1}{2}}$

Ⓐ $10(\frac{1}{2})+3z=1$
$3z=-4$ $\boxed{z=-\frac{4}{3}}$

① $2(\frac{1}{2})+14y+9(-\frac{4}{3})=-$
$1+14y-12=-7$
$14y=4$
$\boxed{y=\frac{2}{7}}$

$\boxed{(\frac{1}{2},\frac{2}{7},-\frac{4}{3})}$

10. Evaluate the following expressions.

(a) $3(4-7i)\cdot(3-i)$

$3(12-4i-21i+7i^2)$
$3(-25i+5)$
$-75i+15$
$\boxed{15-75i}$

(b) $\dfrac{(1+2i)^2}{5-2i}$

$\dfrac{1+2i+2i+4i^2}{5-2i}$

$\dfrac{-3+4i}{5-2i}$

$\dfrac{(-3+4i)(5+2i)}{(5-2i)(5+2i)}$

$\dfrac{-15-6i+20i+8i^2}{25-4i^2}$

$\boxed{\dfrac{-23+14i}{29}}$

(c) $(1-i)^{20}$

$((1-i)^4)^5=(((1-i)^2)^2)^5$

$(1-i)^2=(1-i)(1-i)$
$=1-2i+i^2$
$=-2i$

$(-2i)^2)^5$
$(4i^2)^5$
$(-4)^5$
$\boxed{-1024}$