$\log x^4 = \log x$ $\log_x 9 = p$

$\log_a x = \frac{1}{p}$

EXER 5I

~~EXER 5I~~

(1) a.) $2^{3x} = 5$

$\ln 2^{3x} = \ln 5$

$3x = \frac{\ln 5}{\ln 2}$

$x = \frac{\ln 5}{3\ln 2}$

$x = \frac{\ln 5}{\ln 8}$

b.) $3^x(3^{x-1}) = 10$

$\log 3^{2x-1} = \log 10$

$2x-1 = \frac{\log 10}{\log 3}$

$2x-1 = \log_3 10$

$x = \frac{\log_3 10 + 1}{2}$

(2.) a) $4\log_3 x = \log_x 3$

$4\frac{\log x}{\log 3} = \frac{\log 3}{\log x}$

$4 = \left(\frac{\log 3}{\log x}\right)^2$

$\pm 2 = \frac{\log 3}{\log x}$

$2 = \log_x 3 \quad -2 = \log_x 3$

$x^2 = 3 \quad x^{-2} = 3$

$x = \sqrt{3} \quad x^2 = \frac{1}{3}$

$x = \frac{1}{\sqrt{3}}$

b.) $3\log_2 x + \log_2 27 = 3$

$\log_2 x^3 + \log_2 27 = 3$

$\log_2 (27x^3) = 3$

$2^3 = 27x^3$

$2^3 = (3x)^3$

$2 = 3x$

$x = 2/3$

(3) $9^x - 6(3^x) - 16 = 0$

$(3^x)^2 - 6(3^x) - 16 = 0$

$(3^x - 8)(3^x + 2) = 0$

$3^x = 8 \quad 3^x = -2$

$x = \frac{\ln 8}{\ln 3} \quad x = \text{NO SOLNS.}$

$x = \log_3 8$

(4) $\log_4 x + 12\log_x 4 - 7 = 0$

$\frac{\log x}{\log 4} + \frac{12\log 4}{\log x} - 7 = 0$

$\frac{\log x}{\log 4} + \frac{\log 4^{12}}{\log x} - 7 = 0$

$(\log x)^2 + 12(\log 4)^2 - 7\log 4 \log x = 0$

$(\log x)^2 - 7\log 4 \log x + 12(\log 4)^2 = 0$

$(\log x - 3\log 4)(\log x - 4\log 4) = 0$

$\log x = 3\log 4 \quad \log x = 4\log 4$

$\log x = \log 64 \quad \log x = \log 256$

$x = 64 \quad x = 256$

$x^2 - 2xy - 3y^2$

$(x - 3y)(x + 1y)$

(5) $5^{x+1} + \frac{4}{5^x} - 21 = 0$ $\cdot 5^x$

$5^{2x+1} + 4 - 21 \cdot 5^x = 0$

$5(5^{2x}) - 21(5)^x + 4 = 0$

$5(5^x)^2 - 21(5^x) + 4 = 0$

$(5(5^x) - 1)(5^x - 4) = 0$

$-5^x \quad -20 \cdot 5^x$

$5(5^x) = 1 \qquad 5^x = 4$

$5^x = \frac{1}{5} \qquad \log_5 5^x = \log_5 4$

$x = -1 \qquad x = \log_5 4$

(6) $\log_3 x + \log_x 9 - 3 = 0$

$\log x \log 3 \left(\frac{\log x}{\log 3} + \frac{2\log 3}{\log x} - 3 = 0\right)$

$(\log x)^2 + 2(\log 3)^2 - 3\log x \log 3 = 0$

$(\log x)^2 - 3\log x \log 3 + 2(\log 3)^2 = 0$

$(\log x - 2\log 3)(\log x - \log 3) = 0$

$\log x = 2\log 3 \qquad \log x = \log 3$

$x = 9 \qquad x = 3$

(7) $3 \cdot 9^x - 2 \cdot 4^x = 5 \cdot 6^x$

$3 \cdot (3^x)^2 - 2 \cdot (2^x)^2 = 5 \cdot 3^x \cdot 2^x$

$3 \cdot (3^x)^2 - 5 \cdot 3^x \cdot 2^x - 2 \cdot (2^x)^2 = 0$

$(3 \cdot 3^x + 1 \cdot 2^x)(1 \cdot 3^x - 2 \cdot 2^x) = 0$

$3 \cdot 3^x = -1 \cdot 2^x \qquad 1 \cdot 3^x = 2 \cdot 2^x$

$\frac{3^x}{2^x} = \frac{-1}{3} \qquad \frac{3^x}{2^x} = \frac{2}{1}$

$\left(\frac{3}{2}\right)^x = -\frac{1}{3} \qquad \left(\frac{3}{2}\right)^x = 2$

NO SOLUTION $\qquad \times \ln \frac{3}{2} = \ln 2$

$x = \frac{\ln 2}{\ln \frac{3}{2}}$

$x = \frac{\ln 2}{\ln 3 - \ln 2}$

1.71

$\frac{\log 2}{\log 1.5}$

$\log_{1.5} 2 = x$

$1.5^x = 2$

$(\sqrt{2}, 4)$ $(181.02, 0.35)$

8.) $6\log_2 x + 6\log_8 y = 7$

$6(\log_2 x + \log_8 y) = 7$

$\frac{\log x}{\log 2} + \frac{\log y}{\log 8} = \frac{7}{6}$

$\left(\frac{\log x}{\log 2} + \frac{\log y}{3\log 2} = \frac{7}{6}\right) \cdot -4$

$3\log x + \log y = \frac{21}{6}\log 2$

$4\log_4 x + 4\log_2 y = 9$

$4(\log_4 x + \log_2 y) = 9$

$\frac{\log x}{\log 4} + \frac{\log y}{\log 2} = \frac{9}{4}$

$\left(\frac{\log x}{2\log 2} + \frac{\log y}{\log 2} = \frac{9}{4}\right) 4\log 2$

$2\log x + 4\log y = 9\log 2$

$-12\log x - 4\log y = -\frac{54}{6}\log 2$

$2\log x + 4\log y = 9\log 2$

$-10\log x = -5\log 2$

$\log x = \frac{1}{2}\log 2$

$\log x = \log 2^{1/2}$

$x = \sqrt{2}$

$4\log_4(\sqrt{2}) + 4\log_2 y = 9$

$\log_4 (2^{1/2})^4$

$\log_4 4$

$1 + 4\log_2 y = 9$

$4\log_2 y = 8$

$\log_2 y = 2$

$y = 2^2$

$y = 4$

$\frac{3\log x}{\log 2} + \frac{\log y}{\log 2} = 21$ $\frac{42}{2}$

$\frac{3\log x}{\log 2} + \frac{2\log y}{\log 2} = \frac{27}{2}$ 13.5

$\frac{5\log y}{\log 2} = \frac{-15}{2}\frac{\log 2}{5}$

$\log y = -\frac{3}{2}\log 2$

$y = 2^{-3/2}$

$(\sqrt{2}, 4)$

$6\log_2 x + 6\log_8 y = 7$

$\frac{6\log y}{3\log 2}$

9.) $2\log_x y = 1 \qquad xy = 125$

$y = \frac{125}{x}$

$2\log_x \frac{125}{x} = 1$

$\log_x \frac{125}{x} = \frac{1}{2}$

$\frac{\log \frac{125}{x}}{\log x} = \frac{1}{2}$

$\frac{\log 125 - \log x}{\log x} = \frac{1}{2}$

$\frac{\log 125}{\log x} - \frac{\log x}{\log x} = \frac{1}{2}$

$\log_x 125 = \frac{3}{2}$

$x^{3/2} = 125$

$x = 125^{2/3}$

$x = \left(\sqrt[3]{125}\right)^2$

$x = 25$

$25y = 125$

$y = 5$

$(25, 5)$

10.) $y\log_2 8 = x \qquad 2^x + 8^y = 64$

$\log_2 2^3 = \frac{x}{y}$

$\frac{x}{y} = 3$

$x = 3y$

$2^x + 2^{3y} = 64$

$2^{3y} + 2^{3y} = 64$

$2 \cdot 2^{3y} = 64$

$2^{3y+1} = 2^6$

$3y + 1 = 6$

$3y = 5$

$y = 5/3$

$x = 3 \cdot \frac{5}{3}$

$x = 5$

$\left(5, \frac{5}{3}\right)$

11.) a) $\log_5 x = y = \log_{25}(2x-1)$

$\log_5 x = \log_{25}(2x-1)$

$\frac{\log x}{\log 5} = \frac{\log 2x-1}{\log 25}$

$\log x \cdot \log 25 = \log(2x-1) \cdot \log 5$

$\log x^2 \log 5 = \log(2x-1) \cdot \log 5$

$\log x^2 = \log(2x-1)$

$x^2 = 2x-1$

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$x = 1$

$y = \log_5 1$

$y = 0$

$5^y = 1$

$y = 0$

(1, 0)

b.) $\log(x+y) = 0$

$x+y = 1$

$x = 1-y$

$2\log x = \log(y+5)$

$2\log(1-y) = \log(y+5)$

$\log(1-y)^2 = \log(y+5)$

$(1-y)^2 = y+5$

$1-2y+y^2 = y+5$

$y^2 - 3y - 4 = 0$

$(y-4)(y+1) = 0$

$y = 4, -1$

$y = 4$

$x = -3$

$x \neq$ NEG

$y = -1$

$x = 2$

(2, -1)