SE P. 251
1-4

(1) a. $5^3 = 125$ b. $10^3 = 1{,}000$ c. $27^{1/3} = 3$

$\log_5 125 = 3$ $\log_{10} 1000 = 3$ $\log_{27} 3 = 1/3$

d. $10^{-3} = 0.001$ e. $m = n^2$ f. $a^b = 2$

$\log_{10} 0.001 = -3$ $\log_n m = 2$ $\log_a 2 = b$

(2) a. $\log_3 9 = 2$ b. $\log_{10} 1{,}000{,}000 = 6$ c. $\log_{49} 7 = \frac{1}{2}$

$3^2 = 9$ $10^6 = 1{,}000{,}000$ $49^{1/2} = 7$

d. $\log_a 1 = 0$ e. $\log_4 a = b$ f. $\log_p q = r$

$a^0 = 1$ $4^b = a$ $p^r = q$

(3) a. $\log_8 64 = x$ b. $\log_9 3 = x$ c. $\log_{10} 0.01 = x$

$8^x = 64$ $9^x = 3$ $10^x = 0.01$

$x = 2$ $x = \frac{1}{2}$ $x = -2$

d. $\log_{144} 12 = x$ e. $\log_{37} 1 = x$ f. $\log_a \sqrt[3]{a} = x$

$144^x = 12$ $37^x = 1$ $a^x = \sqrt[3]{a}$

$x = \frac{1}{2}$ $x = 0$ $x = \frac{1}{3}$

(4) a. $\log_x 81 = 2$ b. $\log_3 x = 4$ c. $\log_{11} 121 = x$

$x^2 = 81$ $3^4 = x$ $11^x = 121$

$x = 9$ $x = 81$ $x = 2$

$\frac{\log 81}{\log x} = 2$ ← $\therefore x \neq -9$!!

d. $\log_x 5 = \frac{1}{3}$ e. $\log_x 16 = \frac{2}{3}$ f. $\log_x 32 = -5$

$x^{1/3} = 5$ $x^{2/3} = 16$ $x^{-5} = 32$

$x = 5^3$ $x = 16^{3/2}$ $x^5 = \frac{1}{32}$

$x = 125$ $x = 64$ $x = \frac{1}{2}$

SF P. 253 #1-4

(1) a. $\log_a \frac{p^2}{q}$

$\log_a p^2 - \log_a q$

$2\log_a p - \log_a q$

b. $\log_a \sqrt[3]{\frac{p}{q^2}} = \log_a \left(\frac{p}{q^2}\right)^{1/3} = \log_a \left(\frac{p^{1/3}}{q^{2/3}}\right)$

$= \log_a p^{1/3} - \log_a q^{2/3}$

$= \frac{1}{3}\log_a p - \frac{2}{3}\log_a q$

(2) a. $\log 4 + 2\log 3 - \log 6$

$\log 2^2 + \log 3^2 - \log 2\cdot 3$

$\log\left(\frac{2^2 \cdot 3^2}{2\cdot 3}\right)$

$\log(2\cdot 3)$

$\log 6$

b. $\frac{1}{2}\log_a p + \frac{1}{4}\log_a q^2$

$\log_a (p)^{1/2} + \log_a (q^2)^{1/4}$

$\log_a p^{1/2} + \log_a q^{1/2}$

$\log_a \sqrt{p} + \log_a \sqrt{q}$

$\log_a \sqrt{pq}$

c. $2 - \log 5$

($\log x = \log_{10} x$, $\log_{10} 10 = 1$) $\longleftrightarrow$ $2\log 10 - \log 5$

$\log 10^2 - \log 5$

$\log\left(\frac{10^2}{5}\right) = \log 20$

(3) a. $\log 5 + \log 8 - \log 4$

$\log\left(\frac{5\cdot 8}{4}\right)$

$\log_{10} 10 = 1$ ($10^1 = 10$)

b. $\log_2 48 - \frac{1}{3}\log_2 27$

$\log_2 48 - \log_2 3$

$\log_2 16 = 4$ ($2^4 = 16$)

c. $2 + \log_5 10 - \log_5 2$

$2\log_5 5 + \log_5 10 - \log_5 2$

$\log_5\left(\frac{5^2 \cdot 10}{2}\right)$

$\log_5 125 = 3$ ($5^3 = 125$)

or $\log_5 10 - \log_5 2$

$\log_5 5 = 1 + 2 = 3$

(4) a. $3\log y = 2\log x$

$\log y^3 = \log x^2$

$y^3 = x^2$

$y = x^{2/3}$ or $y = \sqrt[3]{x^2}$

b. $\log y = \log x + \log 2$

$\log y = \log 2x$

$y = 2x$

c. $\log y - 3\log x = \log 2$

$\log y - \log x^3 = \log 2$

$\log\left(\frac{y}{x^3}\right) = \log 2$

$\frac{y}{x^3} = 2$

$y = 2x^3$

d. $\log y = 2 + 3x$

$y = 10^{2+3x}$

or $\log y = 2\log 10 + 3x \log 10$

$\log y = \log 10^2 + \log 10^{3x}$

$= \log 10^{2+3x}$

EXER 5G

p. 254

1.) a.) $\log_3 2 \times \log_2 81$

$\log_3 2 \times \frac{\log_3 81}{\log_3 2}$

$\log_3 81$

4

b.) $\log_6 10 \times \log 6$

$\log_6 10 \times \frac{\log_6 6}{\log_6 10}$

$\log_6 6$

1

c.) $\log_{125} 8 \times \log_8 5$

$\frac{\log 8}{\log 125} \times \frac{\log 5}{\log 8}$

$\frac{\log 5}{\log 125}$

$\log_{125} 5 \qquad 125^x = 5$

$\frac{1}{3}$

d.) $\frac{1}{\log_2 6} + \frac{1}{\log_3 6}$

$\frac{1}{\frac{\log 6}{\log 2}} + \frac{1}{\frac{\log 6}{\log 3}}$

$\frac{\log 2}{\log 6} + \frac{\log 3}{\log 6}$

$\frac{\log 2 + \log 3}{\log 6} \qquad \frac{\log 6}{\log 6} = 1$

e.) $\frac{1}{\log_4 6} + \frac{1}{\log_9 6}$

$\frac{\log 4}{\log 6} + \frac{\log 9}{\log 6}$

$\frac{\log 36}{\log 6} = 2$

f.) $\frac{\log 40}{\log 5} - \frac{\log 8}{\log 5}$

$\frac{\log 5}{\log 5} = 1$

② a.) $a^{\log b} = b^{\log a}$

$\log a^{\log b} = \log b^{\log a}$

$\log b \cdot \log a = \log b \cdot \log a$

b.) $\frac{1}{\log_a ab} + \frac{1}{\log_b ab}$

$\frac{\log a}{\log ab} + \frac{\log b}{\log ab}$

$\frac{\log a + \log b}{\log ab}$

$\frac{\log ab}{\log ab} = 1$

③ $p = \log_a x \Rightarrow a^p = x$

$q = \log_a y$

$p = \frac{\log x}{\log a}$

(a.) $\log_x a = \frac{\log a}{\log x}$

$\log_x a = \frac{1}{p}$ $\log_y a = \frac{1}{q}$

$\log_{xy} a = \frac{\log_a a}{\log_a xy}$

$= \frac{\log_a a}{\log_a x + \log_a y}$

$= \frac{1}{\frac{1}{p} + \frac{1}{q}}$

$\log_{xy} a = \frac{1}{p+q}$

(b.) $\log_{\frac{x}{y}} a = \frac{1}{p-q}$

$= \frac{\log_a a}{\log_a \frac{x}{y}}$

$= \frac{\log_a a}{\log_a x - \log_a y}$

$\log_{\frac{x}{y}} a = \frac{1}{p-q}$

$\frac{\log 12}{\log 4}$ $\quad p = \log_a x \quad p = \frac{\log x}{\log a} \quad \log_x q = \frac{1}{p}$

Exer. Set: ① a. $5^x = 7$

$\ln 5^x = \ln 7$

$x = \frac{\ln 7}{\ln 5}$

b. $4^{2x-1} = 3$

$\ln 4^{2x-1} = \ln 3$

$(2x-1)\ln 4 = \ln 3$

$2x - 1 = \frac{\log_4 3}{\log 4} + 1$

$\frac{\log_4 3 + 1}{2}$

p. 256

#1-2

$\log_4 12$

$\log_4 (3 \cdot 4)$

$\log_4 3 + \log_4 4$

$\log_4 3 \quad 1$

② $(2^x)(5^x) = 0.01$

$10^x = 0.01$

$x = -2$