Math HL – Year 1 Name: KEY

Functions and Relations – Chapter 2 Review Period: ______

1. Given functions $f(x) = 2x + 1$ and $g(x) = x^3$, find the function $(f^{-1} \circ g)^{-1}$.

$f(x) = 2x+1$

$y = 2x+1$

$x = 2y+1$

$2y = x-1$

$y = \frac{x-1}{2}$

$\boxed{f^{-1}(x) = \frac{x-1}{2}}$

$(f^{-1} \circ g)(x) = f^{-1}(g(x))$

$= f^{-1}(x^3)$

$\boxed{(f^{-1} \circ g)(x) = \frac{x^3-1}{2}}$

$y = \frac{x^3-1}{2}$

$x = \frac{y^3-1}{2}$

$2x = y^3 - 1$

$y^3 = 2x+1$

$\boxed{y = \sqrt[3]{2x+1}}$

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2. A function is called self-inverse if $f(x) = f^{-1}(x)$ for all x in the domain.

(a) Show that $f(x) = \frac{1}{x}, x \neq 0$ is a self-inverse function.

$y = \frac{1}{x} \rightarrow x = \frac{1}{y} \rightarrow y = \frac{1}{x}$ ✓

(b) Find the value of the constant k so that $g(x) = \frac{3x-5}{x+k}, x \neq -k$ is a self-inverse function.

$y = \frac{3x-5}{x+k}$

$x = \frac{3y-5}{y+k}$

$x(y+k) = 3y-5$

$xy + kx = 3y-5$

$xy - 3y = -kx - 5$

$y(x-3) = -kx-5$

$y = \frac{-kx-5}{x-3}$

$\frac{-kx-5}{x-3} = \frac{3x-5}{x+k}$

NUMERATORS =

$-kx - 5 = 3x - 5$

$-kx = 3x$

$\boxed{k = -3}$

DENOMINATORS =

$x - 3 = x + k$

$\boxed{k = -3}$

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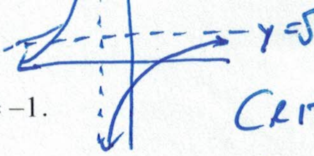
3. Consider the functions given below.

$f(x) = 2x + 3$ and $g(x) = \frac{1}{x}, x \neq 0$

(a) Find $(g \circ f)(x)$ and write down the domain of the function. $g(f(x)) = \frac{1}{2x+3}$ Domain: $x \neq -\frac{3}{2}$

(b) Find $(f \circ g)(x)$ and write down the domain of the function. $f(g(x)) = \frac{2}{x} + 3$ Domain: $x \neq 0$

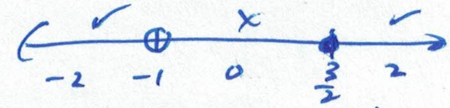


4. Functions g and h are defined by $g(x) = \sqrt{x}$ and $h(x) = \dfrac{2x-3}{x+1}, x \neq -1$.

CRITICAL VALUES

(a) Find the range of h. H.A. @ $y = 2$

(EQUAL POWERS) RANGE: $y \neq 2$ $x = -1$



(b) Solve the equation $h(x) = 0$.

$\dfrac{2x+3}{x+1} \Rightarrow$ $2x - 3 = 0$ $\boxed{x = \frac{3}{2}}$

$\dfrac{2x-3}{x+1} = 0$

$\frac{-7}{-1} \geq 0$ $\quad -\frac{3}{1} \geq 0 \quad \frac{1}{3} \geq 0$

RANGE IS HARD!!

(c) Find the domain and range of $g \circ h$.

$2x - 3 = 0$

DOMAIN: $x < -1,\ x \geq \frac{3}{2}$

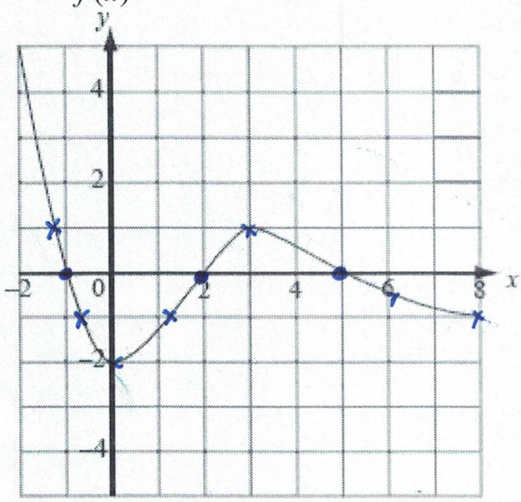
$g(h(x)) = \sqrt{\dfrac{2x-3}{x+1}}$ $\quad \dfrac{2x-3}{x+1} \geq 0$ NO $\sqrt{-}$ $\quad \boxed{x = \frac{3}{2}}$

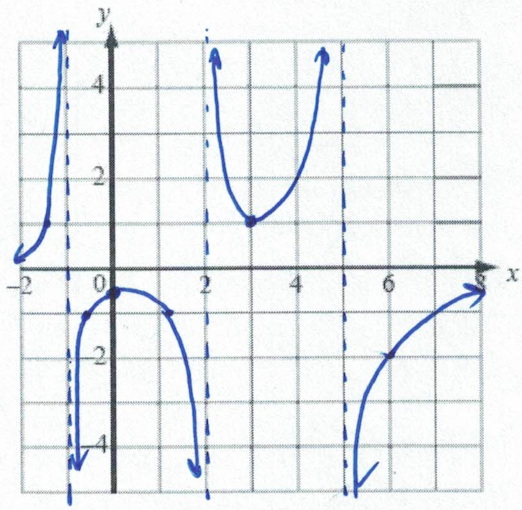
$x \neq -1$

RANGE: $y \geq 0,\ y \neq \sqrt{2}$

HARD!

5. The graph of $y = f(x)$ for $-2 \leq x \leq 8$ is shown. On the set of axes provided, sketch the graph of $y = \dfrac{1}{f(x)}$, clearly showing any asymptotes and indicating the any maximum or minimum values.





6. Let $f(x) = \dfrac{1-x}{1+x}$ and $g(x) = \sqrt{x+1}$, $x > -1$.

Find the set of values of x for which $f(x) \leq g(x)$.

$\dfrac{1-x}{1+x} \leq \sqrt{x+1}$

SQUARE BOTH SIDES

$\dfrac{1-x}{1+x} = \sqrt{x+1}$

$\dfrac{(1-x)^2}{(1+x)^2} = x+1$

$(1-x)^2 = (x+1)^3$

$1 - 2x + x^2 = x^3 + 3x^2 + 3x + 1$

$x(x^2 + 2x + 5) = 0$

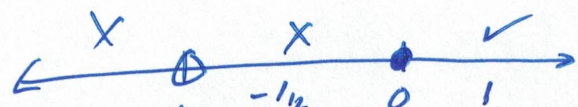
$x = 0$ $\quad x^2 + 2x + 5 = 0$

$x = \dfrac{-2 \pm \sqrt{4 - 4(1)(5)}}{2}$

NO REAL SOLUTIONS

CRITICAL VALUES: $x = 0,\ x = -1$

FROM SOLVING DOMAIN ERROR



DOMAIN ERROR, NO NEED TO TEST

$3 \leq \sqrt{\frac{1}{2}}$ $\quad 0 \leq 1$

$\boxed{x \geq 0}$

7. Let $g(x)=x+1$ and $f(x)=\dfrac{4x}{x-2}, x\neq 2$. If $h(x)=(f\circ g)(x)$, find

(a) $h(x)$;

$h(x)=f(g(x))$

$f(g(x))=\dfrac{4(x+1)}{(x+1)-2}=\boxed{\dfrac{4x+4}{x-1}}$

(b) $h^{-1}(x)$.

$y=\dfrac{4x+4}{x-1}$

$x=\dfrac{4y+4}{y-1}$

$x(y-1)=4y+4$

$xy-x=4y+4$

$xy-4y=x+4$

$y(x-4)=x+4$

$\boxed{y=\dfrac{x+4}{x-4}}$

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8. Let $f(x)=\sqrt{x+4}, x\geq -4$ and $g(x)=x^2, x\in\mathbb{R}$.

(a) Find $(g\circ f)(3)$.

(b) Find $f^{-1}(x)$.

(c) Write down the domain and range of f^{-1}.

ONLY POSITIVE SIDE

(a) $(g\circ f)(3)=g(f(3))$

$=g(\sqrt{7})$

$=\boxed{7}$

(b) $y=\sqrt{x+4}$

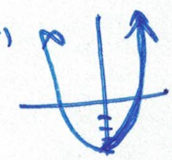
$x=\sqrt{y+4}$

$x^2=y+4$

$y=x^2-4$

$\boxed{f^{-1}(x)=x^2-4}$

(c)



DOMAIN: $x\geq 0$

RANGE: $y\geq -4$

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9. State the domain of the function $f(x)=\dfrac{x^2-9}{\sqrt{x}-9}$.

DENOM $\neq 0$

$\sqrt{x}-9\neq 0$

$\sqrt{x}\neq 9$

$x\neq 81$

NO $\sqrt{-}$

$x\geq 0$

$\boxed{\text{DOMAIN: } x\geq 0,\ x\neq 81}$

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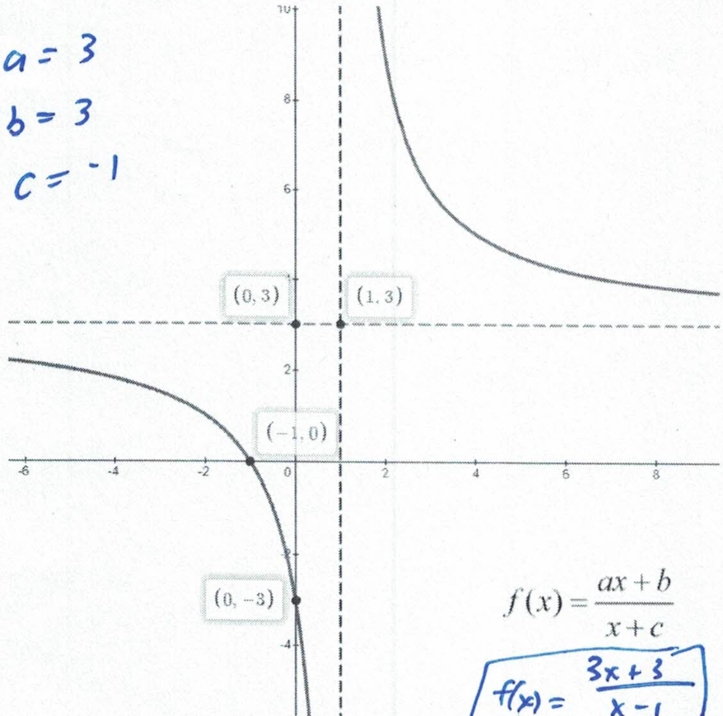
10. The graph of $f(x)=\dfrac{ax+b}{x+c}$ is shown below. Find the values of $a,b,c\in\mathbb{R}$.

$a=3$

$b=3$

$c=-1$



$$f(x)=\frac{ax+b}{x+c}$$

$$\boxed{f(x)=\frac{3x+3}{x-1}}$$

HORIZ. ASYM. AT $y=3$

LEADING COEFF. $\left(\dfrac{a}{1}=3\right.$ $\boxed{a=3}$

VERTICAL ASYM. AT $x=1$

DENOM $=0$

$x+c=0$

$1+c=0$

$\boxed{c=-1}$

Y-INT AT $(0,-3)$

$x=0 \rightarrow \dfrac{b}{c}=-3$ $\quad \dfrac{b}{-1}=-3$ $\boxed{b=3}$

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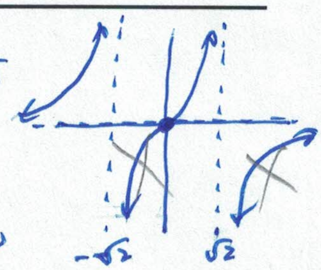
11. State the domain of the function $g(x)=\sqrt{\dfrac{2x}{2-x^2}}$.

$y=\dfrac{2x}{2-x^2}$



CRITICAL VALUES: SET $=0$ AND ANY DOMAIN ERRORS

$\dfrac{2x}{2-x^2}\geq 0$

$\dfrac{2x}{2-x^2}\geq 0$

$\dfrac{2x}{2-x^2}=0$

$2x=0$

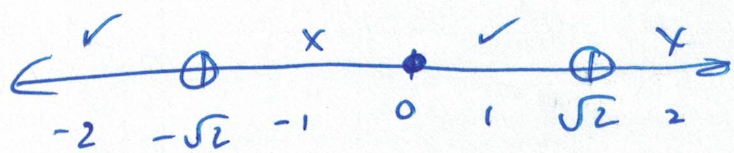
$\boxed{x=0}$

CRITICAL VALUES

$2-x^2\neq 0$

$x^2\neq 2$

$\boxed{x\neq\pm\sqrt{2}}$



$\dfrac{-4}{-2}$ $\quad \dfrac{-2}{1}$ $\quad \dfrac{2}{1}$ $\quad \dfrac{4}{-2}$

$+$ $\quad -$ $\quad +$ $\quad -$

$\boxed{\text{DOMAIN: } x<-\sqrt{2},\ 0\leq x<\sqrt{2}}$