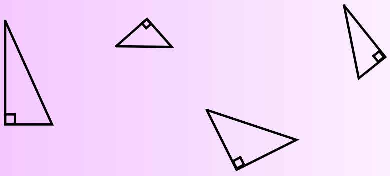
9.1 The Pythagorean Theorem

Intro: What is a "right" triangle?

Definition: A *right triangle* is any triangle that contains a right angle, denoted by a square.

Ex:



Question: What's so special about right triangles?

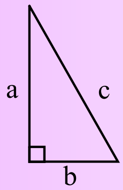
They from the basis of an entire field of mathematics: Trigonometry

Why do we care? One main theorem:

The Pythagorean Theorem

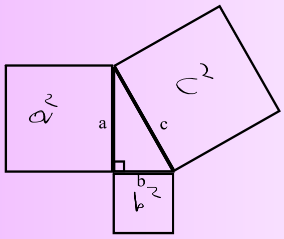
Definition: The Pythagorean Theorem

Given any right triangle, the square of the length of the longest side (the hypotenuse) is equal to the sum of the squares of the lengths of the shorter sides (the legs).



$$a^2 + b^2 = c^2$$

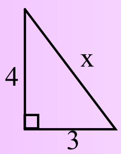
Question: Why is the Pythagorean Theorem true?



What are the areas of these three sqaures?

Question: What's the purpose of the Pythagorean Theorem?

1. Finding side lengths in a right triangle.



What is the length of x?

$3^2 + 4^2 = x^2$

$9 + 16 = x^2$

$25 = x^2$

$5 = \pm x$

But we can ignore negatives, cause there can never be a negative side length. So $x = 5$.

Side Note: Pythagorean Triples

A *Pythagorean Triple* is a set of three positive integers that satisfy the Pythagorean Theorem.

Common Pythagorean Triples and Some of Their Multiples

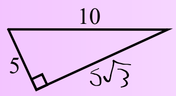
3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
$3x, 4x, 5x$	$5x, 12x, 13x$	$8x, 15x, 17x$	$7x, 24x, 25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold-faced triple by the same factor.

9.1 The Pythagorean Theorem

However: Not all right triangles have integer side lengths.

Find the length of the missing side in the triangle below. Write your answer as a simplified radical.



$10^2 = 5^2 + b^2$

$100 = 25 + b^2$

$75 = b^2$

$b = \sqrt{75}$

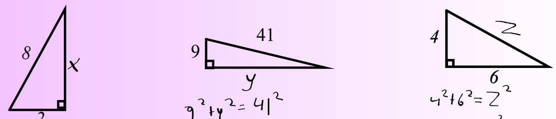
$\sqrt{75}$

$\sqrt{25} \cdot \sqrt{3}$

$5 \cdot \sqrt{3}$

Your Turn!

Find the length of the missing sides. Write your answer as a simplified radical, if necessary.



$8^2 = 2^2 + x^2$

$64 = 4 + x^2$

$60 = x^2$

$x = \sqrt{60} = 2\sqrt{15}$

$9^2 + y^2 = 41^2$

$81 + y^2 = 1681$

$y^2 = 1600$

$y = 40$

$4^2 + 6^2 = z^2$

$16 + 36 = z^2$

$52 = z^2$

$z = \sqrt{52}$

$2\sqrt{13}$

Pythagorean Triples - All sides are positive integers

Big 4! (Most common)

3-4-5	*5-12-13*	*8-15-17*	*7-24-25*
$3^2 + 4^2 = 5^2$	$5^2 + 12^2 = 13^2$	$8^2 + 15^2 = 17^2$	$7^2 + 24^2 = 25^2$
$9 + 16 = 25$	$25 + 144 = 169$	$64 + 225 = 289$	$49 + 576 = 625$
$25 = 25$	$169 = 169$	$289 = 289$	$625 = 625$

Others

9-40-41 11-60-61

20-21-29 12-35-37

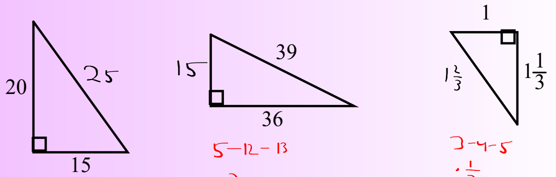
Families of Pythagorean Triples

Multiples or divisors of common triples (scale factors!)

3-4-5	***5-12-13***	***8-15-17***	***7-24-25***
6-8-10 ×2	10-24-26 ×2	1-$\frac{15}{8}$-$\frac{17}{8}$ $\times\frac{1}{8}$	1-$\frac{24}{7}$-$\frac{25}{7}$ $\times\frac{1}{7}$
9-12-15 ×3	60-120-130 ×10	32-60-68 ×4	$7\sqrt{2}$-$24\sqrt{2}$-$25\sqrt{2}$ $\times\sqrt{2}$

Your Turn!

Find the lengths of the missing sides **WITHOUT** using the Pythagorean Theorem. (Hint: triples!)



3-4-5

·5

5-12-13

·3

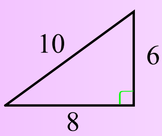
3-4-5

$\cdot\frac{1}{3}$

Question: What's the purpose of the Pythagorean Theorem?

2. Determining if a triangle is a right triangle.

Theorem: If the side lengths of a triangle satisfy the Pythagorean Theorem, then the triangle is a right triangle, where the angle opposite the longest side is a right angle.



$6^2 + 8^2 = 10^2$

$36 + 64 = 100$

$100 = 100$

✓

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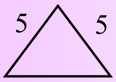
Side Note: The Pythagorean Inequality

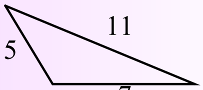
Theorem: Given any triangle, where *c* is the longest side while *a* and *b* are the shorter sides, if...

$a^2 + b^2 > c^2$



Then the triangle is acute.

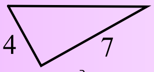
$a^2 + b^2 < c^2$



Then the triangle is obtuse.

Back to You!

Determine if the following triangles are right triangles. If not, determine if they are acute or obtuse.

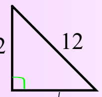


$4^2+7^2 \quad 8^2$

$16+49 \quad 64$

$63 < 64$

obtuse

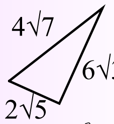


$(6\sqrt{2})^2+(6\sqrt{2})^2 \quad 12^2$

$36\cdot 2+36\cdot 2 \quad 144$

$72+72=144$

✓



$(2\sqrt{5})^2+(6\sqrt{3})^2 \quad (4\sqrt{7})^2$

$4\cdot 5+36\cdot 3 \quad 16\cdot 7$

$20+108 \quad 112$

$128>112$

Acute

Homework:

9.1: #3-6, 12, 21-28, 41