$$\cos 2A = \begin{cases} \cos^2\theta - \sin^2\theta \\ 2\cos^2\theta - 1 \\ 1 - 2\sin^2\theta \end{cases}$$

SE P.402 (CONT.)

4. $\tan\left(\theta + \frac{\pi}{3}\right)\tan\left(\theta - \frac{\pi}{3}\right) = \frac{\tan^2\theta - 3}{1 - 3\tan^2\theta}$

$$= \left(\frac{\tan\theta + \tan\frac{\pi}{3}}{1 - \tan\theta\tan\frac{\pi}{3}}\right)\left(\frac{\tan\theta - \tan\frac{\pi}{3}}{1 + \tan\theta\tan\frac{\pi}{3}}\right)$$

$$\boxed{\tan\frac{\pi}{3} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}}$$

$$= \frac{\tan^2\theta - (\sqrt{3})^2}{1 - \tan^2\theta(\sqrt{3})^2}$$

$$= \frac{\tan^2\theta - 3}{1 - 3\tan^2\theta}$$

$$\cos^2\theta = \frac{\cos 2A + 1}{2}$$

5. a. $2\cos^2 A + \sin^2 A$

$$2\cos^2 A - 1 + 1 + \cos^2\theta - \cos 2A$$

$$2\cos^2 A - 1 + 1 + \frac{\cos 2A + 1}{2} - \cos 2A$$

$\sin^2\theta = \cos^2\theta - \cos 2A$

$$\cancel{\cos 2A} + \cancel{1} + \frac{\cos 2A + 1}{2} - \cancel{\cos 2A}$$

$$\frac{\cos 2A + 3}{2}$$

$$\sin^2\theta = \frac{1 - \cos 2A}{2}$$

b. $\cos^4 A = (\cos^2 A)^2$

$$= \left(\frac{\cos 2A + 1}{2}\right)^2$$

$$= \frac{\cos^2 2A + 2\cos 2A + 1}{4}$$

c. $\sin^4 A = (\sin^2 A)^2$

$$= \left(\frac{1 - \cos 2A}{2}\right)^2$$

$$= \frac{1 - 2\cos 2A + \cos^2 2A}{4}$$