8.4 - Similarity Theorems

Mental Floss: Thurs, Feb 15th

In the diagram, $\triangle ABC \sim \triangle DEF$. Find the area of $\triangle DEF$.
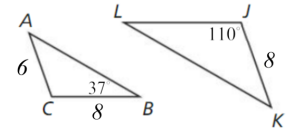
B
10 cm
A   C
E
5 cm
D   F

Area of $\triangle ABC = 36$ cm²

$$\frac{4}{1} = \frac{36}{A}$$
$$4A = 36$$
$$\boxed{A = 9 \text{ cm}^2}$$

RATIO SIDES = 10:5 or 2:1
"   PERIM = 2:1
"   AREAS = 4:1 $(2^2 : 1^2)$

In the diagram, $\triangle ABC \sim \triangle KLJ$. Find the following measures, writing your answer on the lines next to each question. Show any necessary work in the space provided at the bottom of the sheet.

1.) $\overline{LJ}$ _____

2.) $m\angle C$ _____

3.) $m\angle K$ _____

4.) Ratio of perimeters of $\triangle ABC$ to $\triangle KLJ$ _____

5.) Ratio of areas of $\triangle KLJ$ to $\triangle ABC$ _____

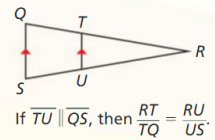A   L   J
110°
6   8
37
C   8   B   K

6.) In the diagram below, find the value of x, the length of $\overline{EG}$.

$$\triangle RAE \sim \triangle AGE$$

A
6
R   3   E   x   G

6
3

x
6

$$\frac{3}{6} = \frac{6}{x}$$
$$3x = 36$$
$$\boxed{x=12}$$

Theorem Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

Q   T   R
S   U

If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.
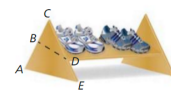

Example #2   Finding the Length of a Segment

In the diagram, $\overline{QS} \parallel \overline{UT}$, $RS = 4$, $ST = 6$, and $QU = 9$. What is the length of $\overline{RQ}$?

$$\frac{x}{9} = \frac{4}{6}$$
$$6x = 36$$
$$\boxed{x=6}$$

R
4
Q   x   S
9   6
U   T

Example #3   Solving a Real-Life Problem

On the shoe rack shown, $BA = 33$ centimeters, $CB = 27$ centimeters, $CD = 44$ centimeters, and $DE = 25$ centimeters. Explain why the shelf is not parallel to the floor.

C
B
A   D
E

27   44
33   25

$$\frac{27}{33} \stackrel{?}{=} \frac{44}{25}$$
NOT EQUAL
$$0.818 \neq 1.76$$

8.4 - Similarity Theorems

Mental Floss: Fri, Feb 16th

In the diagram below, $\triangle ABC \sim \triangle DEC$. Find the following:

a.) $m\angle A$ $38°$

b.) $\overline{CD}$ $\boxed{4}$ $\frac{6}{15} = \frac{x}{10}$ $x = 4$ $15x = 60$

c.) Perimeter of $\triangle DEC$ $\boxed{P = 18}$ $(6 + 4 + 8)$

d.) Ratio of the area of $\triangle DEC$ to $\triangle ABC$ $\boxed{4:25}$ $2:5$

e.) Scale factor from $\triangle ABC$ to $\triangle DEC$ $\boxed{\frac{2}{5}}$ or $\frac{5}{2}$ SMALLER

C.) $\frac{6}{15} = \frac{y}{20}$ $15y = 120$ $y = 8$

Diagram: A — 20 — B, angle $38°$ at A, 55 at B, sides 10 and 15, $87°$ at C, 6, $55°$ at E, 8 to D. Top small triangle D, $38°$, 55, y, x, 67, 6, E, C.
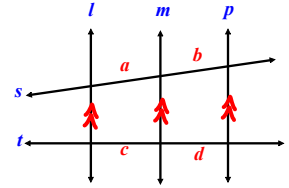

Theorem: Three Parallel Lines Theorem

If three or more parallel lines intersect two transversals, then they divide the transversals proportionally.

$\ell \parallel m \parallel p$

$\frac{a}{c} = \frac{b}{d}$ OR $\frac{c}{d} = \frac{a}{b}$

$ad = bc$

Diagram: lines l, m, p with transversals s, t; segments a, b, c, d.

Example #4 — Using the Three Parallel Lines Theorem

In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are all congruent, $GF = 120$ yards, $DE = 150$ yards, and $CD = 300$ yards. Find the distance HF between Main Street and South Main Street.

120 150 $\frac{120}{x} = \frac{150}{300}$
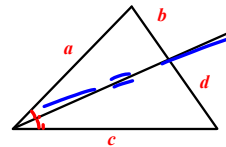
x 240 300 $150x = 36,000$ $x = 240$

$\boxed{HF = 360 \text{ yd}}$

Diagram: F — Main St. — 1 — E; 120 yd, 2, 150 yd; G — D Second St.; 300 yd; 3; H — C — South Main St.

Theorem: Triangle Angle Bisector Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

$\frac{a}{c} = \frac{b}{d}$ or $\frac{b}{a} = \frac{d}{c}$

Diagram: triangle with sides a, b, c, d.

Example #5 — Using the Triangle Angle Bisector Theorem

In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find the length of $\overline{RS}$.

$\frac{7}{13} = \frac{15-x}{x}$

$7x = 13(15-x)$

$7x = 195 - 13x$

$20x = 195$

$\boxed{x = 9.75}$

$\begin{array}{r} 15 \\ \times 13 \\ \hline 45 \\ 150 \\ \hline 195 \end{array}$

$\overline{RS} = 9.75$

Diagram: Q, 7, P, $15-x$, R, 15, 13, x, S.

Homework

Homework

6.4 p.333
  #11-16,21

8.4 p.450
  #4,7,8,13-16,18,21-26,29,30,34,43-45