Conditional Probability

For two events A and B the probability of A occurring given that B has occurred can be found using the equation:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$P(A \mid B)$: Probability of A **given** B

$P(A \cap B)$: Probability of A **and** B

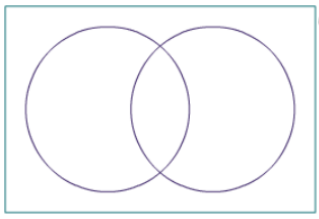
$P(B)$: Probability of B

This formula leads to one of the more common probability formulas used, which we will address later on.

Conditional Probability

Exercise 6F *Modified*

1 In a class of 25 students, 15 study French, 13 study Malay and 5 study neither language.
One of these students is chosen at random.



$P(M) =$

$P(F \cap M) =$

$P(F \mid M) =$

$P(M \mid F) =$

Mr. Urbanc rolls a fair, 6-sided dice. Let A and B be defined as:

A = obtain a score of at least 4

B = obtain an even score

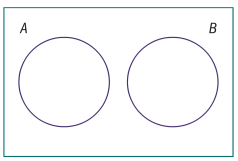
a.) Write down P(A) and P(B).

b.) Draw a tree diagram to illustrate all the possibilities.

Use it to show $\mathrm{P}(A \cap B) = \mathrm{P}(A|B) \times \mathrm{P}(B)$.

Mutually Exclusive Events

If two events are mutually exclusive, then they cannot occur at the same time. Thus, $P(A \cap B) = \varnothing$.



A and B are mutually exclusive.

A and A' are mutually exclusive.

B and B' are mutually exclusive.

IndepenInInIndependent Events

If two events are independent, the probability of the two events occurring simultaneously is equal to the product of the individual probabilities.

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ Formula for Conditional Probability

$P(A \cap B) = P(A \mid B) \cdot P(B)$ Rearrange the equation above

$\boxed{P(A \cap B) = P(A) \cdot P(B)}$ The probability of A is unaffected by B

Multiplication Property for Independent Events

Independent Events

Extensions

$P(A \mid B) = P(A)$
$P(A \mid B') = P(A)$
$P(A' \mid B) = P(A')$
$P(A' \mid B') = P(A')$

The probabilities of A and A' are unaffected by any of the givens, in these cases B and B'.

$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$ 3 independent events

Example #1

Independent events A and B are such that $P(B) = 0.4$ and $P(A \cup B) = 0.75$. Find these probabilities:

a $P(A)$

b $P(A \cap B')$

Homework

- 6G p.305 #1-3
- 6J p.315 #1,3-5,10
- 6K p.320 #4,5,8,9