Maclaurin series $f(x)=\frac{f(0)}{0!}x^0+\frac{f'(0)}{1!}x^1+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\ldots$

Example: Maclaurin expansion for $f(x) = \cos x$.

$$f(x)=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}-\ldots$$

$f(x)=1+\boxed{R_1(x)}$ → 1st degree Maclaurin polynomial

$f(x)=1-\frac{x^2}{2!}+\boxed{R_2(x)}$ → 2nd degree Maclaurin polynomial

$f(x)=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\boxed{R_3(x)}$ → 3rd degree Maclaurin polynomial

$f(x)=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\boxed{R_4(x)}$ → 4th degree Maclaurin polynomial

Error terms

Maclaurin series for special functions

$$e^x=1+x+\frac{x^2}{2!}+\ldots$$

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\ldots$$

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots$$

$$\arctan x=x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots$$

Power Series for Elementary Functions

	Interval of Convergence
$\frac{1}{x}=1-(x-1)+(x-1)^2-(x-1)^3+(x-1)^4-\ldots+(-1)^n(x-1)^n+\ldots$	$0<x<2$
$\frac{1}{1+x}=1-x+x^2-x^3+x^4-x^5+\ldots+(-1)^n x^n+\ldots$	$-1<x<1$
$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\ldots+(-1)^{n+1}\cdot\frac{x^n}{n}$	$-1<x<1$
$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}+\ldots+\frac{x^n}{n!}+\ldots$ $\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\ldots$	$-\infty<x<\infty$
$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\frac{x^9}{9!}-\ldots+\frac{(-1)^n x^{2n+1}}{(2n+1)!}+\ldots$	$-\infty<x<\infty$
$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}-\ldots+\frac{(-1)^n x^{2n}}{(2n)!}+\ldots$	$-\infty<x<\infty$
$\arctan x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}-\ldots+\frac{(-1)^n x^{2n+1}}{2n+1}+\ldots$	$-1\le x\le 1$
$\arcsin x=x+\frac{x^3}{2\cdot 3}+\frac{1\cdot 3x^5}{2\cdot 3\cdot 5}+\frac{1\cdot 3\cdot 5x^7}{2\cdot 4\cdot 6\cdot 7}+\ldots+\frac{(2n)!x^{2n+1}}{(2^n n!)^2(2n+1)}+\ldots$	$-1\le x\le 1$
$(1+x)^k=1+kx+\frac{k(k-1)x^2}{2!}+\frac{k(k-1)(k-2)x^3}{3!}+\frac{k(k-1)(k-2)(k-3)x^4}{4!}+\ldots$	$-1<x<1^*$

The convergence at $x=\pm 1$ depends on the value of k.

Composite Maclaurin series

1.) Find the series expansion of $f(x)=\cos(2x^3)$.

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots$$

$$\cos(2x^3)=1-\frac{(2x^3)^2}{2}+\frac{(2x^3)^4}{24}-\ldots$$

$$\cos(2x^3)=1-\frac{1}{2}(4x^6)+\frac{1}{24}(16x^{12})-\ldots$$

$$\boxed{\cos(2x^3)=1-2x^6+\frac{2}{3}x^{12}-\ldots}$$

2.) Write the first 5 non-zero terms of the Maclaurin expansion for $g(x)=\sin x^2$.

$$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots$$

$$\sin x^2=(x^2)-\frac{(x^2)^3}{6}+\frac{(x^2)^5}{120}-\ldots$$

$$\boxed{\sin x^2=x^2-\frac{1}{6}x^6+\frac{1}{120}x^{10}-\ldots}$$

3.) Find the series expansion of $f(x)=x\cdot\cos(x^3)$ as far as the x^4 term.

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots$$

$$\cos x^3=1-\frac{1}{2}x^6+\frac{1}{24}x^{12}-\ldots$$

$$\boxed{x\cdot\cos x^3=x-\frac{1}{2}x^7+\frac{1}{24}x^{13}-\ldots}$$

4.) Find the 1st 4 terms of the Maclaurin series for $f(x)=e^x \arctan x$.

$e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots$

$\arctan x = x-\frac{x^3}{3}+\frac{x^5}{5}-\dots$

OTHER TERMS BIGGER THAN WE NEED...

$e^x\cdot\arctan x = \left(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3\right)\left(x-\frac{1}{3}x^3+\frac{1}{5}x^5\right)$

$= x-\frac{1}{3}x^3+\cancel{\frac{1}{5}x^5}$

$x^2-\frac{1}{3}x^4+\cancel{\frac{1}{5}x^6}$

$\frac{1}{2}x^3\cancel{-\frac{1}{6}x^5}+\cancel{\frac{1}{10}x^7}$

$\frac{1}{6}x^4\cancel{-\frac{1}{18}x^6}+\cancel{\frac{1}{30}x^8}$

TOO BIG... ONLY NEED 1st 4 TERMS (x, x^2, x^3, x^4)

$\boxed{e^x\cdot\arctan x = x+x^2+\frac{1}{6}x^3-\frac{1}{6}x^4}$

5.) Use a power series to approximate $\int_0^1 e^{-x^2}dx$. Use the first four terms of the series for your approximation.

$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots$

$e^{-x^2} = 1-x^2+\frac{1}{2}x^4-\frac{1}{6}x^6+\dots$

$\int_0^1 e^{-x^2}dx = \int_0^1\left(1-x^2+\frac{1}{2}x^4-\frac{1}{6}x^6\right)dx$

$= \left[x-\frac{1}{3}x^3+\frac{1}{10}x^5-\frac{1}{42}x^7\right]_0^1$

$= \left(1-\frac{1}{3}+\frac{1}{10}-\frac{1}{42}\right)-(0)$

$\boxed{\int_0^1 e^{-x^2}dx = \frac{26}{35}}$

6.) Find the series expansion of $h(x)=e^{\sin x}$ as far as the x^2 term.

$e^x = 1+x+\frac{x^2}{2!}+\dots$

$\sin x = 1-\frac{x^3}{3!}+\dots$

$e^{\sin x} = 1+\left(1-\frac{x^3}{3!}\right)+\frac{\left(1-\frac{x^3}{3!}\right)^2}{2!}$

$= 1+1-\frac{1}{6}x^3+$

CHANGE THIS!!

1. Find the first four non-zero terms of the Maclaurin series for:
(a) (i) $\boxed{\sin(3x^4)}$ (ii) $\cos(2\sqrt{x})$
(b) (i) $\ln(2+3x)$ (ii) $\ln(1-2x)$
(c) (i) $e^{-\frac{x^2}{2}}$ (ii) e^{x^3}

$\sin(3x^4) = (3x^4)-\frac{(3x^4)^3}{3!}+\frac{(3x^4)^5}{5!}-\frac{(3x^4)^7}{7!}$

$= 3x^4-\frac{27x^{12}}{6}+\frac{243x^5}{120}-\frac{2187x^7}{5040}$

$= \boxed{3x^4-\frac{9}{2}x^{12}+\frac{81}{40}x^{20}-\frac{243}{560}x^{28}}$

1. Find the first four non-zero terms of the Maclaurin series for:
(a) (i) $\sin(3x^4)$ (ii) $\boxed{\cos(2\sqrt{x})}$
(b) (i) $\ln(2+3x)$ (ii) $\ln(1-2x)$
(c) (i) $e^{-\frac{x^2}{2}}$ (ii) e^{x^3}

$\cos(2\sqrt{x}) = 1-\frac{(2\sqrt{x})^2}{2!}+\frac{(2\sqrt{x})^4}{4!}-\frac{(2\sqrt{x})^6}{6!}$

$= 1-\frac{4x}{2}+\frac{16x^2}{24}-\frac{64x^6}{720}$

$= \boxed{1-2x+\frac{2}{3}x^2-\frac{4}{45}x^3}$

1. Find the first four non-zero terms of the Maclaurin series for:
(a) (i) $\sin(3x^4)$ (ii) $\cos(2\sqrt{x})$
(b) (i) $\boxed{\ln(2+3x)}$ (ii) $\ln(1-2x)$
(c) (i) $e^{-\frac{x^2}{2}}$ (ii) e^{x^3}

$\ln(2+3x) = \ln\left(2\left(1+\frac{3}{2}x\right)\right) = \ln 2+\ln\left(1+\frac{3}{2}x\right)$

$= \ln 2+\frac{3}{2}x-\frac{\left(\frac{3}{2}\right)^2x^2}{2}+\frac{\left(\frac{3}{2}\right)^3x^3}{3}$

$= \boxed{\ln 2+\frac{3}{2}x-\frac{9}{8}x^2+\frac{9}{8}x^3}$

1. Find the first four non-zero terms of the Maclaurin series for:

(a) (i) $\sin(3x^4)$ (ii) $\cos(2\sqrt{x})$

(b) (i) $\ln(2+3x)$ (ii) $\boxed{\ln(1-2x)}$

(c) (i) $e^{-\frac{x^2}{2}}$ (ii) e^{x^3}

$$\ln(1-2x) = \ln(1+(-2x))$$
$$= (-2x) - \frac{(-2x)^2}{2} + \frac{(-2x)^3}{3} - \frac{(-2x)^4}{4}$$
$$= \boxed{-2x - 2x^2 - \frac{8}{3}x^3 - 4x^4}$$

1. Find the first four non-zero terms of the Maclaurin series for:

(a) (i) $\sin(3x^4)$ (ii) $\cos(2\sqrt{x})$

(b) (i) $\ln(2+3x)$ (ii) $\ln(1-2x)$

(c) (i) $\boxed{e^{-\frac{x^2}{2}}}$ (ii) e^{x^3}

$$e^{-\frac{x^2}{2}} = 1 + \left(-\frac{x^2}{2}\right) + \frac{\left(-\frac{x^2}{2}\right)^2}{2!} + \frac{\left(-\frac{x^2}{2}\right)^3}{3!}$$
$$= \boxed{1 - \frac{x^2}{2} + \frac{x^4}{8} - \frac{x^6}{48}}$$

1. Find the first four non-zero terms of the Maclaurin series for:

(a) (i) $\sin(3x^4)$ (ii) $\cos(2\sqrt{x})$

(b) (i) $\ln(2+3x)$ (ii) $\ln(1-2x)$

(c) (i) $e^{-\frac{x^2}{2}}$ (ii) $\boxed{e^{x^3}}$

$$e^{x^3} = 1 + (x^3) + \frac{(x^3)^2}{2!} + \frac{(x^3)^3}{3!}$$
$$= \boxed{1 + x^3 + \frac{x^6}{2} + \frac{x^9}{6}}$$

2. By combining Maclaurin series of different functions find the series expansion as far as the term in x^4 for:

(a) (i) $\boxed{\ln(1+x)\sin 2x}$ (ii) $\ln(1-x)\cos 3x$

(b) (i) $\frac{e^x}{1+x}$ (ii) $\frac{\sin x}{1-2x}$

(c) (i) $\ln(1+\sin x)$ (ii) $\ln(1-\sin x)$

$$\ln(1+x)\sin 2x = \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}\right)\left((2x) - \frac{(2x)^3}{3!}\right)$$
$$= \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}\right)\left(2x - \frac{4x^3}{3}\right)$$
$$= 2x^2 - x^3 + \frac{2}{3}x^4 - \cancel{\frac{1}{2}x^5} - \frac{4}{3}x^4 + \cancel{\frac{2}{3}x^5} - \cancel{\frac{4}{9}x^6} + \cancel{\frac{1}{3}x^7}$$
$$= \boxed{2x^2 - x^3 - \frac{2}{3}x^4}$$

2. By combining Maclaurin series of different functions find the series expansion as far as the term in x^4 for:

(a) (i) $\ln(1+x)\sin 2x$ (ii) $\boxed{\ln(1-x)\cos 3x}$

(b) (i) $\frac{e^x}{1+x}$ (ii) $\frac{\sin x}{1-2x}$

(c) (i) $\ln(1+\sin x)$ (ii) $\ln(1-\sin x)$

$$\ln(1-x)\cos 3x = \left((-x) - \frac{(-x)^2}{2} + \frac{(-x)^3}{3} - \frac{(-x)^4}{4}\right)\left(1 - \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!}\right)$$
$$= \left(-x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4\right)\left(1 - \frac{9}{2}x^2 + \frac{27}{8}x^4\right)$$
$$= \cancel{-x} - \cancel{\frac{1}{2}x^2} - \cancel{\frac{1}{3}x^3} - \frac{1}{4}x^4 + \cancel{\frac{9}{2}x^3} + \frac{9}{4}x^4 + \cancel{\frac{3}{2}x^4}$$
$$= \boxed{-x - \frac{1}{2}x^2 + \frac{25}{6}x^3 + 2x^4}$$

2. By combining Maclaurin series of different functions find the series expansion as far as the term in x^4 for:

(a) (i) $\ln(1+x)\sin 2x$ (ii) $\ln(1-x)\cos 3x$

(b) (i) $\boxed{\frac{e^x}{1+x}}$ (ii) $\frac{\sin x}{1-2x}$

(c) (i) $\ln(1+\sin x)$ (ii) $\ln(1-\sin x)$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$$
$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \ldots$$
$$e^x \cdot \frac{1}{1+x} = \left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4\right)\left(1 - x + x^2 - x^3 + x^4\right)$$
$$= (\cancel{1} - \cancel{x} + \cancel{x^2} - \cancel{x^3} + \cancel{x^4}) + (\cancel{x} - \cancel{x^2} + \cancel{x^3} - \cancel{x^4}) + \left(\cancel{\frac{1}{2}x^2} - \frac{1}{2}x^3 + \frac{1}{2}x^4\right) + \left(\frac{1}{6}x^3 - \frac{1}{6}x^4\right) + \left(\frac{1}{24}x^4\right)$$
$$= \boxed{1 + \frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{3}{8}x^4}$$

2. By combining Maclaurin series of different functions find the series expansion as far as the term in x^4 for:

(a) (i) $\ln(1+x)\sin 2x$ (ii) $\ln(1-x)\cos 3x$

(b) (i) $\dfrac{e^x}{1+x}$ (ii) $\boxed{\dfrac{\sin x}{1-2x}}$

(c) (i) $\ln(1+\sin x)$ (ii) $\ln(1-\sin x)$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$\ln(1-2x) = \ln(1+(-2x)) = (-2x) - \frac{(-2x)^2}{2} + \frac{(-2x)^3}{3} - \frac{(-2x)^4}{4} = -2x - 2x^2 - \frac{8}{3}x^3 - \ldots$

$\frac{1}{1-2x} = -\frac{1}{2}(-2 - 4x - 8x^2 - 16x^3) = 1 + 2x + 4x^2 + 8x^3$

$\sin x \cdot \frac{1}{1-2x} = \left(x - \frac{x^3}{6}\right)(1 + 2x + 4x^2 + 8x^3)$

$= (x + 2x^2 + 4x^3 + 8x^4) + \left(-\frac{1}{6}x^3 - \frac{1}{3}x^4 \sim\right)$

$= \boxed{x + 2x^2 + \frac{23}{6}x^3 + \frac{23}{3}x^4}$

2. By combining Maclaurin series of different functions find the series expansion as far as the term in x^4 for:

(a) (i) $\ln(1+x)\sin 2x$ (ii) $\ln(1-x)\cos 3x$

(b) (i) $\dfrac{e^x}{1+x}$ (ii) $\dfrac{\sin x}{1-2x}$

(c) (i) $\boxed{\ln(1+\sin x)}$ (ii) $\ln(1-\sin x)$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$

$\ln(1+\sin x) = \ln\left(1 + \left(x - \frac{x^3}{3!}\right)\right)$

$= \left(x - \frac{x^3}{3!}\right) - \frac{1}{2}\left(x - \frac{x^3}{3!}\right)^2 + \frac{1}{3}\left(x - \frac{x^3}{3!}\right)^3 - \frac{1}{4}\left(x - \frac{x^3}{3!}\right)^4$

$= \left(x - \frac{x^3}{6}\right) - \frac{1}{2}\left(x^2 - \frac{1}{3}x^4 + \cancel{\frac{x^6}{36}}\right) + \frac{1}{3}(x^3 - \sim) - \frac{1}{4}(x^4 \sim)$

$= x - \frac{1}{6}x^3 - \frac{1}{2}x^2 + \frac{1}{6}x^4 + \frac{1}{3}x^3 - \frac{1}{4}x^4$

$= \boxed{x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4}$

2. By combining Maclaurin series of different functions find the series expansion as far as the term in x^4 for:

(a) (i) $\ln(1+x)\sin 2x$ (ii) $\ln(1-x)\cos 3x$

(b) (i) $\dfrac{e^x}{1+x}$ (ii) $\dfrac{\sin x}{1-2x}$

(c) (i) $\ln(1+\sin x)$ (ii) $\boxed{\ln(1-\sin x)}$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$

$\sin x = x - \frac{x^3}{3!} + \ldots$

$\ln(1-\sin x) = \ln(1+(-\sin x))$

$= \ln\left(1 + \left(-x + \frac{x^3}{3!}\right)\right)$

$= \left(-x + \frac{x^3}{3!}\right) - \frac{1}{2}\left(-x + \frac{x^3}{3!}\right)^2 + \frac{1}{3}\left(-x + \frac{x^3}{3!}\right)^3 - \frac{1}{4}\left(-x + \frac{x^4}{4!}\right)^4$

SEE ABOVE

$= \left(-x + \frac{x^3}{3!}\right) - \frac{1}{2}\left(x^2 - \frac{1}{3}x^4 + \cancel{\frac{x^6}{36}}\right) + \frac{1}{3}(-x^3 \sim) - \frac{1}{4}(x^4 \sim)$

$= -\cancel{x} + \cancel{\frac{1}{6}x^3} - \cancel{\frac{1}{2}x^2} + \frac{1}{6}x^4 - \cancel{\frac{1}{3}x^3} - \frac{1}{4}x^4$

$= \boxed{-x - \frac{1}{2}x^2 - \frac{1}{6}x - \frac{1}{12}x^4}$