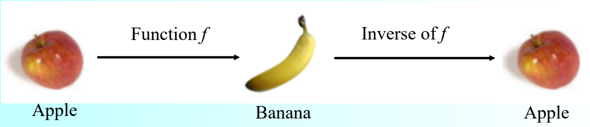
2I - Inverse Functions

What do **inverse functions** do?



$+ | -$ $\quad \times | \div$ $\quad x^2 / \sqrt{x}$ $\quad \sin$ / $\sin^{-1}$

Inverse functions undo the operations of each other. See the example below, where *f(x)* and *g(x)* are inverse functions of each other.

Choose any real number value for the **input** of the function *f*.

$\boxed{6}$ Input $\rightarrow$ $f(x) = 2x + 3$, $2(6)+3$ $\rightarrow$ 15 $\rightarrow$ $g(x) = \frac{x-3}{2}$, $\frac{15-3}{2}$ $\rightarrow$ $\boxed{6}$ Output

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Inverse functions

Input x $\rightarrow$ $f(x)$ $\rightarrow$? $\rightarrow$ $g(x)$ $\rightarrow$ Output x

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Inverse Functions

f(x) and *g(x)* are *inverse functions* if the following is true:

$$(f \circ g)(x) = (g \circ f)(x) = x$$

$$f(g(x)) = g(f(x)) = x$$

Inverse Function Notation

Although you have been working with inverse operations for years, inverse functions became more prominent when learning about **trig functions.**

Trig		Inverse Trig
$\sin x$	$\sin^2 x \rightarrow (\sin x)^2$	$\sin^{-1} x$ =
$\cos x$		$\cos^{-1} x$ =
$\tan x$		$\tan^{-1} x$ =

Notation of Inverse Functions

Inverse functions have similar notation to exponents from Algebra 1, so some confusion may exist.

Reciprocal	Inverse Function
$x^{-2} = \frac{1}{x^2}$	$f^{-1}(x)$ or f^{-1}

Note: The index $^{-1}$ when applied to *numbers or variables* means the reciprocal of a number. When applied to *functions* it means the inverse function.

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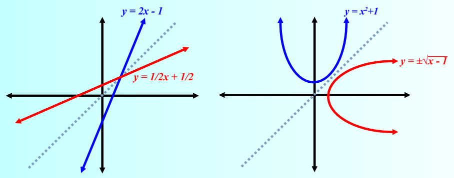
Graphical Interpretation of Inverse Functions

The graphs of *inverse functions* are reflections over the line $y = x$. Below are two examples.



For *inverse functions* the domain and range flip! The domain of a function becomes the range of its inverse and vice versa!

Algebraic Interpretation of Inverse Functions

To find the inverse of a function, perform the steps below:

1. Write the function as y in terms of x ($y = ...$)
2. Swap the x and y values
   (Every y becomes an x and every x becomes a y)
3. Solve for y
4. Apply any restrictions from the original function.

Note: Only functions that are one-to-one will have inverse functions with no restrictions.

Example #1

Find the inverse of the function below. Confirm your results graphically.

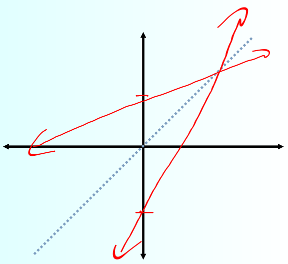
$$f(x) = \frac{x}{2} + 5$$

$y = \frac{x}{2} + 5$

$x = \frac{y}{2} + 5$

$x - 5 = \frac{y}{2}$

$\boxed{2x - 10 = y}$



Example #2

Find the inverse of the function below.

D: $x \in \mathbb{R}, x \neq 3$

R: $y \in \mathbb{R}, y \neq -1$

$$g(x) = \frac{1+x}{3-x}$$

$y = \frac{1+x}{3-x}$

$\boxed{x = \frac{1+y}{3-y}}$

$x(3-y) = 1+y$

$3x - xy = 1 + y$

$-xy - y = 1 - 3x$

$y(-x-1) = 1 - 3x$

$\boxed{y = \frac{1-3x}{-x-1}}$

D: $x \neq -1$

R: $y \neq 3$

D + R FLIP!