Evaluating Improper Integrals $\int_a^\infty f(x)dx$

Divergent

- Area under the curve ***cannot*** be approximated with a finite value. Why?
  - Function was increasing
  - Function ***does not*** get smaller fast enough
- How do you know?
  - Integrate function and answer = ∞
  - Power of x $\left(\frac{1}{x^p}, p \le 1\right)$
  - Comparison Test

    $0 \le f(x) \le g(x)$

Evaluating Improper Integrals $\int_a^\infty f(x)dx$

Convergent

- Area under the curve ***can*** be approximated with a finite value. Why?
  - Function ***does*** get smaller fast enough
- How do you know?
  - Integrate function and get a non-∞ number
  - Power of x $\left(\frac{1}{x^p}, p > 1\right)$
  - Comparison Test

    $0 \le f(x) \le g(x)$

If an integral is convergent, then:

- What is the actual area under the curve?
- If you integrate function, your answer is the area
- When using the power of x, $\left(\frac{1}{x^p}, p > 1\right)$ you should be able to integrate if necessary to find the area even when you did not need it to prove convergence.
- Comparison Test - How?

  $0 \le f(x) \le g(x)$

Review of Infinite Series

64,16,4,1,...

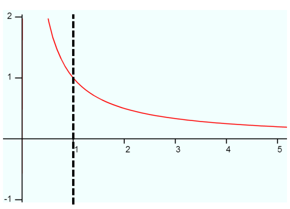
1.) Find the 20th term in the sequence.

2.) Find the sum of the first 12 terms of the sequence.

3.) Find the sum to infinity of the sequence.

Upper and Lower Riemann Sums

Decreasing Functions

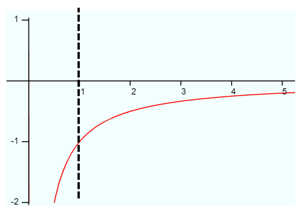


Upper Sums

Lower Sums

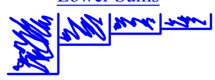
Upper and Lower Riemann Sums

Decreasing Functions

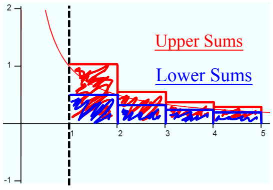


Upper Sums

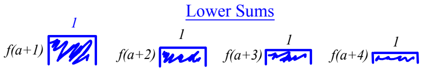
Lower Sums



Decreasing Functions



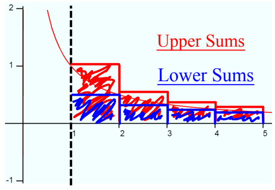
$$A_{\text{rectangle}} = l \cdot w$$

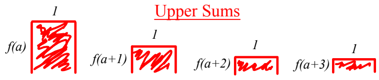


$$A_{\text{total}} = \quad 1\cdot f(a+1) \quad 1\cdot f(a+2) \quad 1\cdot f(a+3) \quad 1\cdot f(a+4)$$

$$\sum_{k=a+1}^{\infty} f(k)$$

Decreasing Functions

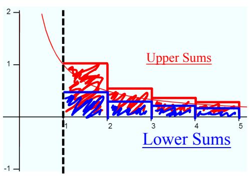




$$A_{\text{total}} = \quad 1\cdot f(a) \quad 1\cdot f(a+1) \quad 1\cdot f(a+2) \quad 1\cdot f(a+3)$$

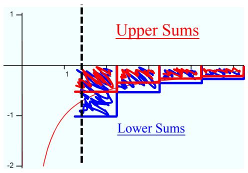
$$\sum_{k=a}^{\infty} f(k)$$

Decreasing Functions



$$\sum_{k=a+1}^{\infty} f(k) < \int_a^{\infty} f(x)dx < \sum_{k=a}^{\infty} f(k)$$

Increasing Functions



$$\sum_{k=a}^{\infty} f(k) < \int_a^{\infty} f(x)dx < \sum_{k=a+1}^{\infty} f(k)$$

Ex1: Find the lower and upper sums (U and L) such that:

$$L < \int_1^{\infty} -\left(\frac{1}{3}\right)^x dx < U$$

Hint: First identify whether the function is increasing or decreasing.

Ex2:
a.) Find the lower and upper sums for $\int_3^{\infty} \frac{1}{x^2}dx$ and write the inequality using summation notation. You do not need to compute the actual values.

b.) Find the actual value of $\int_3^{\infty} \frac{1}{x^2}dx$.