Continuity

A function $f(x)$ is said to be continuous

at some point $x=c$ if $\lim_{x \to c} f(x) = f(c)$.

3 Conditions for Continuity

1.) $f(x)$ must be defined at point c (c is part of the domain of $f(x)$)

2.) The limit of $f(x)$ exists at c

$$\lim_{x \to c^-} = \lim_{x \to c^+} = \lim_{x \to c}$$
$$LHL = RHL$$

3.) The limit of $f(x)$ at c is equal to the function value at c

$$\lim_{x \to c} f(x) = f(c)$$

Differentiability

For a function $f(x)$ to be differentiable at some point $x=c$,

1.) $f(x)$ must be continuous at c

3 Conditions for Continuity		
$f(x)$ exists	$\lim_{x \to c^-} = \lim_{x \to c^+} = \lim_{x \to c}$ $LHL = RHL$	$\lim_{x \to c} f(x) = f(c)$

2.) $f(x)$ must not have a "sharp point" at c

3.) The tangent to $f(x)$ at c must not be vertical

⭐ Based on #1, differentiability implies continuity!

But, continuity does not necessary imply differentiability.
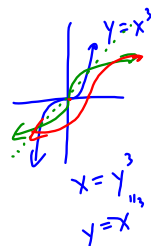
Differentiability

Determine whether the function is differentiable at the given point.

Example 1: $f(x) = (x-1)^{\frac{1}{3}}$ at $x=1$

1.) CONTINUOUS $\left(\begin{array}{l} f(1) = 0 \\ \lim_{x \to 1} = 0 \\ (0=0) \end{array} \right)$ ✓

2.) $f'(x) = \frac{1}{3}(x-1)^{-2/3}$ ✓

3) $f'(1) = $ UNI. ✗

Differentiability

Determine whether the function is differentiable at the given point.

Example 2: $g(x) = |x|$ at $x=0$

1.) CONT. $\longrightarrow \left(\begin{array}{l} -f(0) = 0 \\ -\lim_{x \to 0} = 0 \\ -0 = 0 \end{array} \right)$

2.) $g(x) = \begin{cases} x \geq 0 & x \\ x < 0 & -x \end{cases}$

$g'(x) = \begin{cases} x \geq 0 & 1 \\ x < 0 & -1 \end{cases}$ SHARP PT. ✗

Differentiability

Find the values of the constants a and b such that the function below is differentiable for all $x > 0$.

Example 3: $h(x) = \begin{cases} \ln x & x \leq 3 \\ ax+b & x > 3 \end{cases}$ $(3, \ln 3)$

1.) CONTINUOUS

$\ln 3 = a(3) + b$

$\ln 3 = \frac{1}{3}(3) + b$

$b = \ln 3 - 1$

$h'(x) = \begin{cases} \frac{1}{x} & x \leq 3 \\ a & x > 3 \end{cases}$

$\frac{1}{x} = a$

$a = \frac{1}{3}$

Homework

1F p.33-34 #1-5,8,9