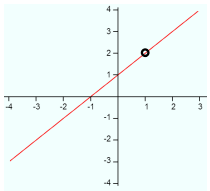
Limit Notation

1.) $g(x)=\dfrac{x^2-1}{x-1}$



Left-hand limit (LHL)

$$\lim_{x\to 1^-} g(x)=$$

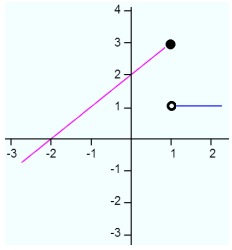
Right-hand limit (RHL)

$$\lim_{x\to 1^+} g(x)=$$

$$\lim_{x\to 1} g(x)=$$

3.)

$$h(x)=\begin{cases} x+2 & x\le 1 \\ 1 & x>1 \end{cases}$$



$$\lim_{x\to 0^-} h(x)=$$

$$\lim_{x\to 2} h(x)=$$

$$\lim_{x\to 1^-} h(x)=$$

$$\lim_{x\to 1^+} h(x)=$$

$$\lim_{x\to 1} h(x)=$$

Limit Notation (Cont.)

The limit, $\boldsymbol{L}$, of a function $\boldsymbol{f}$ exists as $\boldsymbol{x}$ approaches a real value $\boldsymbol{c}$ is defined as:

$$\lim_{x\to c} f(x)=L \Leftrightarrow \left(\lim_{x\to c^-} f(x)=L \text{ and } \lim_{x\to c^+} f(x)=L\right), L\in\mathbb{R}$$

If LHL = RHL = L and L is a real #, then the limit exists.

Using a GDC, sketch the graph of each function and find the limit, if it exists.

1 $\lim\limits_{x\to -1}\dfrac{x^2-1}{x+1}$

2 $\lim\limits_{x\to 1}\dfrac{x^3-1}{x-1}$

3 $\lim\limits_{x\to 2}\begin{cases} 3x-1, & x<2 \\ \dfrac{1}{x^2-1}, & x\ge 2 \end{cases}$

4 $\lim\limits_{x\to 0}\dfrac{|x|}{x}$

5 $\lim\limits_{x\to 6}(x-6)^{\frac{2}{3}}$

6 $\lim\limits_{x\to 3}\lfloor x\rfloor$

1 Find the required limit algebraically, if it exists.

a $\lim\limits_{x\to 4}\left(\dfrac{x+3}{x-3}\right)$

b $\lim\limits_{x\to -2}\left(\dfrac{x^2+x-2}{x+2}\right)$

c $\lim\limits_{x\to -2}\left(\dfrac{x^6-64}{x^3-8}\right)$

d $\lim\limits_{x\to 0}\dfrac{x^2-1}{x^2-x}$

e $\lim\limits_{x\to 1}\dfrac{x^2-1}{x^2-x}$

f $\lim\limits_{x\to 1}\dfrac{1}{1+\dfrac{1}{1-x}}$

g $\lim\limits_{x\to 0}\dfrac{(2+3x)^2-4(1+x)^2}{6x}$

h $\lim\limits_{x\to a}\dfrac{x^2-a^2}{x-a}$

Limit Examples

1.) $\lim\limits_{x\to 0}\left(\dfrac{(x+2)^2}{\sqrt{x^2+4x+13}-3}\right)$

2.) $\lim\limits_{x\to 0} x\cdot\cos\left(\dfrac{1}{x}\right)$

l'Hopital's Rule

Given functions *f(x)* and *g(x)* such that either:

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0} \quad \text{or} \quad \lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\infty}{\infty}$$

then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Note:
Limits must exist

Example: $\lim_{x\to\infty}\frac{x^2+7x-3}{x^2+10x+20}$

Limit Examples

3.) $\lim_{x\to 0}\frac{\sin x}{x}$

4.) $\lim_{x\to e}\frac{1-\ln x}{x-e}$

5.) $\lim_{x\to\infty}\frac{\ln x}{x}$

6.) $\lim_{x\to\infty}\frac{e^x}{x^2}$

Limit Examples

7.) $\lim_{x\to 0^+} x\ln x$

Homework

Limits
1C p.18-19 #2,4,5,7 (#6 for practice)

l'Hopital's Rule
1D p.23-24 #1-6