1B - Warmups

$$\lim_{a\to\infty}\frac{6a+3}{a^2-7}$$ $\frac{\frac{1}{a^2}}{\frac{1}{a^2}}$ $= \frac{\frac{6}{a}+\frac{3}{a^2}}{1-\frac{7}{a^2}} = \frac{0}{1} = 0$

$$\lim_{n\to\infty}\frac{1}{n^2} = 0$$ Bottom Heavy

$$\lim_{n\to\infty}\frac{n^2}{\sqrt{n^4+1}}$$ $\frac{\frac{1}{\sqrt{n^4}}\left(\frac{1}{n^2}\right)}{\frac{1}{\sqrt{n^4}}} = \frac{1}{\sqrt{1+\frac{1}{n^4}}} = \frac{1}{1} = 1$

1B - Squeeze Theorem

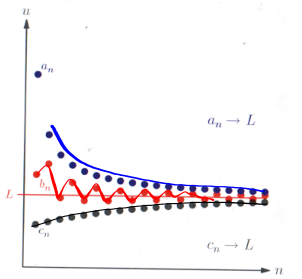
If we have sequences a_n, b_n, and c_n such that:

$a_n \le b_n \le c_n$ for all $n \in \mathbb{Z}^+$

and

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L < \infty$

Then $\lim_{n\to\infty} b_n = L$



1B - Squeeze Theorem - Examples

1.) $$\lim_{n\to\infty}\frac{\sin n}{n}$$

$-1 \le \sin n \le 1$

$-\frac{1}{n} \le \frac{\sin n}{n} \le \frac{1}{n}$

$\lim_{n\to\infty} -\frac{1}{n} = 0$

$\lim_{n\to\infty} \frac{1}{n} = 0$

$\therefore \lim_{n\to\infty} \frac{\sin n}{n} = 0$ by Sq. Thm.

1B - Squeeze Theorem - Examples

2.) $$\lim_{n\to\infty}\frac{2-\cos n}{n+3}$$

$-1 \le \cos n \le 1$ (·by -1)

$-1 \le -\cos n \le 1$

$1 \le 2-\cos n \le 3$

$\frac{1}{n+3} \le \frac{2-\cos n}{n+3} \le \frac{3}{n+3}$

$\lim_{n\to\infty} \frac{1}{n+3} = \lim_{n\to\infty} \frac{3}{n+3} = 0$ (Bottom Heavy)

$\therefore \lim_{n\to\infty} \frac{2-\cos n}{n+3} = 0$

1B - Squeeze Theorem - Examples

3.) $\lim_{n\to\infty} \frac{n!}{n^n}$

$\lim_{n\to\infty} \frac{n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot 2\cdot 1}{n\cdot n\cdot n\cdot\ldots\cdot n\cdot n}$ — n TERMS IN BOTH NUM. & DENOM.

$\frac{n!}{n^n}$ IS ALWAYS ≥ 0

$0 \leq \frac{n!}{n^n} \leq ?$

$0 \leq \frac{n!}{n^n} \leq \frac{1}{n}$

$\lim_{n\to\infty} \frac{1}{n} = 0$

$\therefore \lim_{n\to\infty} \frac{n!}{n^n} = 0$

1B - Squeeze Theorem - Examples

4.) Problem #2 from 1B homework

a.) Show that $\frac{6^n}{n!} \leq \frac{6^5}{5!} \times \frac{6}{n}$ for $n \geq 6$.

$\frac{6\cdot 6\cdot 6\cdot\ldots\cdot 6\cdot 6\cdot 6\cdot 6}{(n-1)(n-2)(n-3)\ldots 5\cdot 4\cdot 3\cdot 2\cdot 1} \leq \frac{6\cdot 6\cdot 6\cdot 6\cdot 6}{5\cdot 4\cdot 3\cdot 2\cdot 1}\cdot\frac{6}{n}$

$\frac{6\cdot 6\cdot\ldots\cdot 6\cdot 6}{(n-1)(n-2)\ldots 7\cdot 6} \leq 1$

NUM $\leq$ DENOM FOR ALL VALUES OF n.

$\therefore \frac{6^n}{n!} \leq \frac{6^5}{5!}\cdot\frac{6}{n}$ $n \geq 6$

Homework

1B p.10-11 #1(a,b,d),3,4

Key Problems: #1(a), 4