17A-17C - Indefinite Integrals

In this chapter, you will learn:

- to reverse the process of differentiation (this process is called integration)
- to find the equation of a curve given its derivative and a point on the curve
- to integrate sin x, cos x, and tan x
- to integrate e^x and $1/x$
- to find the area between a curve and the x- or y-axis
- to find the area enclosed between two curves

How easy are things to undo/reverse?

- Walking down the hallway
- Driving to school
- Mixing two decks of cards together
- Making/baking a cake

Integral Calculus

Integration $\longrightarrow$ Antidifferentiation

Integration is the inverse of differentiation

Differentiation	Integration
$f(x) = x^2 + 7$	$f'(x) = 2x$
$f'(x) = 2x$	$f(x) = x^2 + c$

Integration

Integrate the following equations, giving your answer as y in terms of x.

$\frac{dy}{dx} = 2x$

$y = x^2 + c$

$\frac{dy}{dx} = 4x^2 - 8x^1 + 9$

$y = \frac{4}{3}x^3 - 4x^2 + 9x + c$

Indefinite Integral

$$\int 2x\,dx = \underbrace{x^2 + c}, c \in \mathbb{R}$$

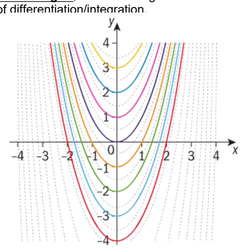
Elongated letter S

dx is always added to an integral to represent the variable you are integrating with respect to

Constant of Integration

Every function has an infinite number of anti-derivatives. This family of functions, known as the ***indefinite integral***, is found using the rules of differentiation/integration

$f(x) = x^2$
$f(x) = x^2 + 5$
$f(x) = x^2 - 1$
$f(x) = x^2 + 10$



Graph of family of curves of $y = x^2 + c$ for different values of c

Indefinite Integral

$f(x)$ is the derivative of $F(x)$

$$\int \underbrace{f(x)}\,dx = F(x) + c, c \in \mathbb{R}$$

Integrand

Variable of integration (with respect to)

Constant of Integration

Note: When listing general rules of integration, the constant "c" is sometimes left off for simplicity

Function	First Derivative
$f(x)$	$f'(x)$
$f'(x)$	$f''(x)$
Uppercase F $\longrightarrow F(x)$	$f(x) \longleftarrow$ Lowercase f

17A-17C - Indefinite Integrals

Finding the derivative

$x^n \longrightarrow$ Multiply the coefficient of x by n $\longrightarrow$ Decrease the exponent of x by 1 $\longrightarrow nx^{n-1}$

Finding the integral (Reverse the process)

$nx^{n-1} \longrightarrow$ Increase the exponent of x by 1 $\longrightarrow$ Divide by the new exponent $\longrightarrow$ Add a constant of integration, c $\longrightarrow x^n + c$

The general rule for integrating x^n for any rational power where $n \neq -1$ is:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, c \in \mathbb{R}$$

Note the condition $n \neq -1$ to avoid division by 0.

Alternate form: $\int x^n dx = \frac{1}{n+1} \cdot x^{n+1} + c$

EXAM HINT
The + c is a part of the answer, and you must write it every time.

To integrate a multiple of a function, we use a process similar to differentiation.

$$\int k \cdot f(x)dx = k \cdot \int f(x)dx$$

EXAM HINT
This rule only works if k is a constant.

Also, since we can differentiate term by term, then we can also split up integrals of sums and differences.

$$\int f(x) \pm g(x)dx = \int f(x)dx \pm \int g(x)dx$$

EXAM HINT
Be warned! You cannot integrate products or quotients by integrating each part separately.

Ex: $\int (x^2 + 7x + 8)dx = \int x^2 dx + \int 7x dx + \int 8 dx$

$$= \frac{1}{3}x^3 + \frac{7}{2}x^2 + 8x + c$$

Worked example 17.1

Find (a) $\int 6x^{-3}\,dx = -3x^{-2} + c$

(b) $\int (3x^4 - 8x^{-\frac{4}{3}} + 2)\,dx = \frac{3}{5}x^5 + 24x^{-\frac{1}{3}} + 2x + c$

Worked example 17.2

Find (a) $\int 5x^2 \sqrt[3]{x}\,dx = \int 5x^2 \cdot x^{1/3}\,dx$

$= \int 5x^{7/3}\,dx$

$= 5 \cdot \frac{3}{10} x^{10/3} + c$

$= \boxed{\frac{3}{2}x^{10/3} + c}$

(b) $\int \frac{(x-3)^2}{\sqrt{x}}\,dx = \int \frac{x^2 - 6x + 9}{x^{1/2}}\,dx$

$= \int x^{3/2} - 6x^{1/2} + 9x^{-1/2}\,dx$

$= \frac{2}{5}x^{5/2} - 6 \cdot \frac{2}{3}x^{3/2} + 9 \cdot 2x^{1/2} + c$

$= \boxed{\frac{2}{5}x^{5/2} - 4x^{3/2} + 18x^{1/2} + c}$

17A p.570 #1
17B p.571 #1-2
17C p.574 #1-4