4 Find the value of a such that the points A(a, $a-1$), B(2, $2a$) and C(0, $3a$) are collinear.

Vector Algebra in 3D

$$u=\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix} \qquad v=\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$$

1.) The sum of vectors u and v is defined by:

$$u+v=\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}+\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}=\begin{pmatrix}u_1+v_1\\u_2+v_2\\u_3+v_3\end{pmatrix}$$

Think of this as adding the x, y, and z components of each vector together.

2.) The product of a scalar λ and a vector u is defined by:

$$u=\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix} \qquad \lambda u=\lambda\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}=\begin{pmatrix}\lambda u_1\\\lambda u_2\\\lambda u_3\end{pmatrix}$$

Think of this as "distributing" the scalar constant to the x, y and z components of the vector.

3.) The *zero vector* (also known as the *null vector*) is:

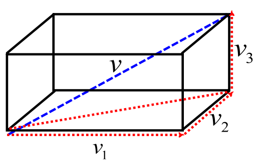
$$0=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

4.) The *opposite vector* of $u=\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}$ is $-u=\begin{pmatrix}-u_1\\-u_2\\-u_3\end{pmatrix}$.

The *magnitude* of vector $v=\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$ is:

$$|v|=\sqrt{(v_1)^2+(v_2)^2+(v_3)^2}$$

If you break down a vector into x, y and z components, think of the magnitude as applying the 3D variation of the Pythagorean Theorem.



$$v^2=(v_1)^2+(v_2)^2+(v_3)^2$$

$$|v|=\sqrt{(v_1)^2+(v_2)^2+(v_3)^2}$$

A *unit vector* is any vector of magnitude 1.

$$i=\begin{pmatrix}1\\0\\0\end{pmatrix} \qquad j=\begin{pmatrix}0\\1\\0\end{pmatrix} \qquad k=\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$i+2j-3k=1\begin{pmatrix}1\\0\\0\end{pmatrix}+2\begin{pmatrix}0\\1\\0\end{pmatrix}-3\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}1\\2\\-3\end{pmatrix}$$

Linear combination of the unit vectors i, j, and k.

Component Form

If $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ and is non-zero, the unit vector in the same direction as v is defined as:

$$u = \frac{1}{|v|}v = \frac{1}{|v|}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

Two vectors are *collinear* if:

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \qquad v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

Then $u = kv$ or $v = ku$ for some scalar k.

If $k>0$ then the vectors have the same direction.

If $k<0$ then the vectors have opposite directions.

To find vectors with a magnitude of m that are collinear to a vector, you can use the formula:

$$w = \pm\frac{m}{|v|}v = \pm\frac{m}{|v|}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

Examples

$$a = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} \qquad b = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$$

1.) Find the magnitude of vector a.

2.) Find the unit vectors collinear with b.

3.) Find the vectors with magnitude equal to b and collinear to a.

Homework

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