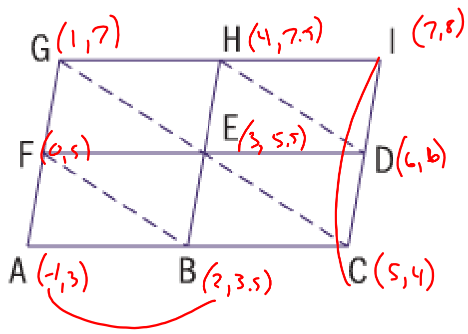
BF
CH
DG



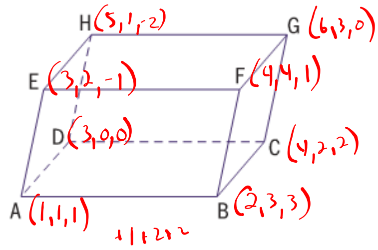
$\vec{AB} = \begin{pmatrix} 3 \\ 0.5 \end{pmatrix}$ $\vec{AE} = \begin{pmatrix} 4 \\ 2.5 \end{pmatrix}$ $\vec{CD} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\vec{BF} = -2i + 1.5j$

3 The diagram shows a parallelepiped ABCDEFGH.

Given $\vec{OA} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\vec{OB} = \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}$, $\vec{OD} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$ and $\vec{OE} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

+2 +1 -2



find in component form:

a $\vec{AB}$ **b** $\vec{AD}$ **c** $\vec{AE}$

d $\vec{AG}$ **e** $\vec{BD}$ **f** $\vec{BH}$

$\begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ -3 \\ -3 \end{pmatrix}$ $\begin{pmatrix} 3 \\ -2 \\ -5 \end{pmatrix}$

Vector Algebra in 2D

$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

1.) The sum of vectors u and v is defined by:

$$u + v = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \end{pmatrix}$$

Think of this as adding the x and y components of each vector together.

Vector Algebra in 2D (Cont.)

$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

2.) The product of a scalar λ and a vector u is defined by:

$$\lambda u = \lambda \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \lambda u_1 \\ \lambda u_2 \end{pmatrix}$$

Think of this as "distributing" the scalar constant to the x and y components of the vector.

Vector Algebra in 2D (Cont.)

$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

3.) The *zero vector* (also known as the *null vector*) is:

$$0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

4.) The *opposite vector* of $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ is $-u = \begin{pmatrix} -u_1 \\ -u_2 \end{pmatrix}$.

Homework

11A p.558 #1-2

11B p.562 #1-4

11C p.564 #1-4

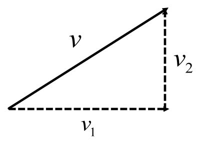
11D p.567 #1,2,4

Vector Algebra in 2D (Cont.)

The *magnitude* of vector $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ is:

$$|v| = \sqrt{(v_1)^2 + (v_2)^2}$$

If you break down a vector into x and y components, think of the magnitude as applying the Pythagorean Theorem.



$$v^2 = (v_1)^2 + (v_2)^2$$

Vector Algebra in 2D (Cont.)

A *unit vector* is any vector of magnitude 1.

$$\underbrace{i = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad j = \begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{\text{Base Unit Vectors in 2D}}$$

Ex1: Show that the following vector is a unit vector.

$$w = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix} \quad |w| = \sqrt{\left(\tfrac{3}{5}\right)^2 + \left(\tfrac{4}{5}\right)^2} = \sqrt{\tfrac{9}{25} + \tfrac{16}{25}} = \boxed{1} \checkmark$$

Vector Algebra in 2D (Cont.)

Finding a *unit vector* in a particular direction is a bit more complicated

Ex2: Find the unit vector in the same direction as the vector below.

$$r = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \quad |r| = \sqrt{5^2 + 12^2} = 13$$

(5-12-13 TRIPLE)

$$u = \frac{1}{13}\begin{pmatrix} 5 \\ 12 \end{pmatrix} = \begin{pmatrix} 5/13 \\ 12/13 \end{pmatrix}$$

SEE NEXT SLIDE FOR FORMULA

Vector Algebra in 2D (Cont.)

If $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ and is non-zero, the unit vector in the same direction as v is defined as:

$$u = \frac{1}{|v|}v \checkmark \quad = \frac{1}{|v|}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \checkmark \quad = \frac{1}{\sqrt{v_1^2 + v_2^2}}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad = \begin{pmatrix} \dfrac{v_1}{\sqrt{v_1^2+v_2^2}} \\ \dfrac{v_2}{\sqrt{v_1^2+v_2^2}} \end{pmatrix}$$

EXCESSIVE!

Vector Algebra in 2D (Cont.)

$$u = \frac{1}{|v|}v$$

Ex3: Find the unit vector in the same direction as the vector below.

$$v = \begin{pmatrix} 2 \\ 7 \end{pmatrix} \quad |v| = \sqrt{2^2 + 7^2} = \sqrt{53}$$

$$u = \frac{1}{\sqrt{53}}\begin{pmatrix} 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 2/\sqrt{53} \\ 7/\sqrt{53} \end{pmatrix}$$

NO NEED TO RATIONALIZE DENOM. UNLESS IT WILL SIMPLIFY

Vector Algebra in 2D (Cont.)

Two vectors are *collinear* if:

$$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \qquad v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

Then $u = kv$ or $v = ku$ for some scalar k.

If $k>0$ then the vectors have the same direction.

If $k<0$ then the vectors have opposite directions.

$k=0$ OK, BUT IT DOES NOT HELP OFTEN

Vector Algebra in 2D (Cont.)

To find vectors with a magnitude of m that are collinear to a vector, you can use the formula:

NEW MAGNITUDE (SCALES IT UP)

ORIGINAL VECTOR

$$u = \pm \frac{m}{|v|} v$$

COLLINEAR +/- (BOTH DIRECTIONS)

ORIG. VECTOR'S MAGNITUDE (MAKES IT A UNIT VECTOR)

Ex4: Find vectors with a magnitude of 3 that are collinear to the vector below:

$$v = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \quad |v| = \sqrt{4^2 + (-3)^2} = 5$$

$$\boxed{u = \pm \frac{3}{5} \begin{pmatrix} 4 \\ -3 \end{pmatrix}}$$

Vector Algebra in 2D (Cont.)

Ex5: Show that the zero vector and **any other** vector are collinear.

$$u = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

Show $u = kv$ or $v = ku$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = k \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} kv_1 \\ kv_2 \end{pmatrix}$$

$k = 0$!!

11E Homework (SUMMARY PAGE)

1.) Find the magnitude of a vector v by applying the variation of the Pythagorean Theorem using the x and y components of v.

$$v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}, |v| = \sqrt{v_1^2 + v_2^2}$$

2.) Divide vector v by the magnitude from #1 to get a unit vector u in the same direction as v.

$$u = \frac{1}{|v|} \cdot v = \frac{1}{|v|} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

3.) Add a +/- symbol to #2 to find both unit vectors that are collinear to v (same ***and*** opposite direction).

$$u = \pm \frac{1}{|v|} \cdot v = \frac{1}{|v|} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

4.) Multiply your answer from #2 by a new magnitude m to find a vector with magnitude m in the same direction as v.

$$u = \frac{m}{|v|} \cdot v = \frac{m}{|v|} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

Homework (✓ on TUES 3/8)

11A p.558 #1-2

11B p.562 #1-4

11C p.564 #1-4

11D p.567 #1,2,4

11E p.570 #1-6